THE UNIVERSITY OF ALBERTA

INDIVIDUALIZED INSTRUCTION IN GRADE SEVEN MATHEMATICS: THE TEACHER'S ROLE

by

(C) E. Gerrit te Kampe

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF EDUCATION

DEPARTMENT OF SECONDARY EDUCATION

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "Individualized Instruction in Grade Seven Mathematics: The Teacher's Role," submitted by B. Gerrit te Kampe, in partial fulfilment of the requirements for the degree of Master of Education.

ABSTRACT

The study was designed to investigate and describe teacher behavior in the individualized instruction method in grade seven mathematics. In order to show the differences in behavior required, a control group was used for the purpose of comparison. There were five teachers in the experimental group and three teachers in the control group. Both groups covered the same material at about the same rate. The teachers in the experimental group used prepared reference material while the teachers in the control group used the authorized text book.

The main purpose of the thesis was to describe what a teacher in the individualized setting did in the classroom and out of the classroom. Suitable instruments had to be designed to gather the data necessary for the description of the teacher behaviors, attitudes, and workload. Three types of instruments were designed by the writer. The instrument used for classroom observation was based on a technique used by Medley and Mitzel. The writer came to this decision after a review of related literature.

The teachers in the experimental group were observed three times each in the independent study class and twice each in the teacher-taught class. The teachers in the control group were observed four times each, except one who was observed only three times. The teachers in both the experimental and the control group were asked to keep the log sheet for an entire week on three different occasions. At the end of the investigation the teachers in both settings were

to complete a questionnaire on their personal reactions.

The findings based on the data obtained from the above instruments showed that the behaviors of the teachers in the experimental groups were considerably different from those of the teachers in the control group. There were two aspects of the experimental setting: the independent study class and the teacher-taught class. The teachers in the independent study class did not check assignments; they did not teach formally; they did not use directed practise; and they did not deal with the entire class extensively. The teachers in the independent study class did spend a considerable part of the class period dealing with individual pupils. The teachers in the independent study class were more positive in their actions than negative. They appeared to have very few discipline problems which turned out to be deceiving, for the teachers felt that it was much more difficult to maintain discipline in the independent study class.

The same teachers conducted the teacher-taught classes. In this class the teachers did spend 100 per cent of the time lecturing and explaining. The pupils were mainly free to attend or to work on their own. Depending on the difficulty of the topic under discussion, the attendance would vary from about 15 to about 55.

The teachers in the experimental group needed considerably more time for marking, preparation, and group planning than the teachers in the control group.

Despite the heavy workload the teachers were generally satisfied with the format of the program and reported to prefer individualized instruction over regular classroom teaching.

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CHAPTER I

THE PROBLEM

Introduction

For many years authors of educational psychology texts and professors of educational methods courses have advocated that the classroom teacher should provide for individual differences. However, little more than lipservice has been paid to this important concept (Glaser, 1966). Some early attempts worthy of mention have been made to provide for individual differences. The Dalton plan and the Winnetka plan, for instance, are probably the best known of these early attempts. Perhaps these plans were too far ahead of their times, for none were widely adopted. Many were never attempted beyond a single school jurisdiction. No one has any concrete evidence as to why these methods did not find wider use. The reasons why the methods did not find adoption could be many and varied. Among some of the reasons that could be listed, perhaps, are the following: lack of communication; lack of financial resources; lack of qualified and competent teaching staff; lack of interest; the possibility that people were not ready for change; and perhaps the fact that some methods just did not work.

Some methods found a much wider application and yet even these slowly disappeared. The Enterprise method was perhaps the best example of this. The Province of Alberta began using this method in 1936

under the leadership of H. C. Newland. The Enterprise method did not remain a permanently established method mainly due to lack of communication (Patterson, 1969). There was no effort made by the innovators to sort out the differences teachers had, for instance, with respect to the idea of permissiveness (freedom) and the idea of activity as these were intended by the originators of the method. The teaching ranks were also divided due to the blurring of the goals by the leaders. The leaders were more concerned about the teaching procedure than about the final outcomes and hence failed to relate ends to their means (Patterson, 1969). Patterson (1969) indicated that the final blow to the Enterprise method was the drain-off of the teaching personnel by the war effort during the second world war. The people called back into the teaching service were unfamiliar with the Enterprise method and coupled with this was a lack of available sources of help. Communication and preparation had been and remained a problem of the Enterprise method. The Enterprise method was introduced in 1936 after Newland had trained only 75 of the best teachers in the province during a summer school in 1935. Hence the teaching staff in the province was poorly prepared when the method was introduced province-wide (Patterson, 1969).

Communication is important especially when new teaching methods are being developed. Presently team-teaching is in vogue. The writer has found, in talking to several teachers, that once again they are reluctant to use team-teaching, mainly, because the teachers are not aware of what is expected of them. The reason for this reluctance is perhaps due to the lack of reporting, by researchers, in the

professional literature, exactly what the teachers did during team-teaching experiments.

The problem cited in relation to team-teaching appears to exist in much of the literature reporting educational innovations. As an example, the writer checked through the issues for the years 1967, 1968, and 1969 of "The Mathematics Teacher" and found that of thirty-five research articles, dealing with innovations in mathematics education, only five articles made reference to what a teacher was expected to do. None of the articles described what the teachers actually did or how much time was required for preparation and related tasks. In general, articles referring to individualized instruction also are lacking in describing the teacher-role. Because the teacher-role aspect, and particularly the teacher-interaction with the pupils, of individualized instruction are so central to its success in the classroom, the writer has decided to study and describe the teacher aspect of individualized instruction in secondary school mathematics and to describe as clearly as possible what a teacher does in this situation: both in and out of the classroom. The particular form of individualized instruction used for the purpose of this study will be described in Chapter IV.

Individualized instruction is one of the latest attempts at providing for individual differences. This teaching method attempts to let each pupil proceed at his own rate and at his own level of achievement. A project based on the concept of individualized instruction had been started by a small group of teachers of grade seven mathematics. The teachers agreed to a more elaborate study than they

were attempting, and so this project was started with their cooperation. Results obtained from this project are being reported in three companion theses: one by Marvin Westrom, describing the development of materials and the theoretical model for this project; one by Audrey Sunde, reporting on pupil achievement; and one by the writer, describing the teacher role in the experimental setting.

Statement of the Problem

The reports on research published in "The Mathematics Teacher", referred to in the introduction, seemed to report most frequently on pupil achievement in a particular program or on how attitude and behavior of pupils changed during the duration of the experiment. They failed to report on what and how teacher behavior changed. To fill a need in this area and because of the importance of the teacher-interaction with the pupils in the individualized setting, the writer decided to do the study. The writer found it necessary to formulate some general questions which would help him in the design of instruments and in research procedures. Specifically, he formulated the following questions:

1. What does a mathematics teacher do in a classroom in which individualized instruction is operating?
2. How much time does he spend on the tasks related to his teaching in the individualized setting?
3. How does he plan and prepare?
4. What are his reactions to the program?
5. How do the above in the individualized instructional setting compare with the same areas for teachers in a

regular instructional setting?

Significance of the Problem

It is important to obtain the answers to the above questions and to describe teacher behavior for three reasons. One, it is important to evaluate the method, for if seemingly impossible tasks are asked of the teachers using this method then the method has inadequacies and is undesirable. Secondly, it is important to describe the behavior in order to be able to replicate the experiment. Thirdly, it is important to describe teacher behavior because of the implications for the training of teachers, administrators, and support staff in the use of this method. In other words it is important because of the implications for the restructuring of method courses which are intended to prepare the teacher to use individualized instruction. Included in this latter point are also the implications for inservice training of the existing personnel.

Limitations

Since the writer was unable to select the teachers used in the project randomly and since the number of teachers was small it was necessary to report the findings as a case study. Thus the results of the study cannot be generalized to any existing larger population.

Delimitations

Since the instruments used were basically designed by the writer and since no facilities nor time were available for properly piloting these instruments, the results of the research are delimited by the adequacy of the instruments used.

Definition of Terms

Individualized Instruction:

Individualized instruction is a style of teaching which permits pupils to progress at their own rate and at their own level through the required materials. The pupil has the freedom to work independently or in cooperation with one or more pupils. The teacher acts as a resource person and diagnostician and makes sure that the pupil works to his full potential.

Regular Classroom Teaching:

Regular classroom teaching is that style of teaching which is in accordance with the presently accepted notion of instruction.

Experimental Group:

The experimental group is that group of teachers involved in the individualized instruction.

Observation Schedule:

The Observation Schedule is the instrument used in the recording of observed classroom behaviors of pupils and teachers. The schedule is in Appendix B.

Log Sheet:

The log sheet is a page on which the teacher daily records the different amounts of time spent on the indicated activities. The log sheet is in Appendix C.

IPI:

IPI, Individually Prescribed Instruction, is a form of individualized instruction developed for elementary schools in the Oakleaf School in Pittsburgh.

Formal Teaching:

Formal teaching is the act of teaching in which the teacher makes a presentation or explanation to the entire class with or without pupil participation.

Teacher Lectures:

Where the teacher gives a discourse for an extended period of time without the verbal participation of the pupils.

Directed Practise:

The teacher presents the pupils with a task related to the subject matter just taught and gives specific instructions. The intention is to find out if the pupils grasped the idea(s) the teacher tried to teach.

Seat Work:

Students are working on assigned material either independently or in small groups at their desks.

Teacher's Role:

The teacher's role is the function and its attendant behaviors which a teacher displays in relation to a required instructional mode.

Outline of the Thesis

In this section the writer intends to give a brief overview of the contents of the thesis. In Chapter II a review of the literature related to the observation techniques and the teacher role in individualized instruction will be given. The development of the instruments used in the gathering of the data for this thesis will be described briefly in Chapter III. Also a brief discussion on the use of the

instruments will be included in this chapter. Next in Chapter IV, the writer will describe the design of the experiment, its procedures, and the analysis of the data, and finally a section on the calculation procedures used for the analysis of the data. Then in Chapter V the results, the interpretations, and conclusions will be reported. The concluding chapter, Chapter VI, will contain the implications of the findings for further research, teacher training, and inservice training. The thesis ends with the Bibliography and the Appendices. In the appendix will appear a sample of the materials used in the individualized instruction program, a sample of each instrument used for the collection of the data and the data used for the analysis of the results in this study.

CHAPTER II

REVIEW OF THE LITERATURE

Introduction

The literature related to the thesis can be divided into two categories. The first category includes the literature related to the teacher role in individualized instruction. The second category includes the literature related to observation techniques used for observing teachers.

Literature Related to Individualized Instruction

As was noted in Chapter I a number of early attempts were made to provide for individual differences. Among these were the Dalton plan and the Winnetka plan. The Dalton plan was first introduced in 1919 and was used mostly from grade four through high school. In 1920 it was adopted by a high school in Dalton, Massachusetts. The major emphasis was on group life rather than on the curriculum itself (Anderson, 1966: 79). The program had two distinct phases. One part contained the academic subjects which were taught on an individualized approach and the other part contained the non-academic subjects which were taught by class or group methods on a non-graded basis. In the academic subjects the pupil contracted a unit of work which he could complete at his own speed. Each contract contained about a month's work. The subjects were subdivided into a number of related jobs or contracts. The pupil had to satisfy one requirement

at all times in that he was to maintain an even pace across all subjects. The writer briefly referred to the organization of the program in order to be able to infer what the teacher might be doing in this setting. The teachers were to be subject specialists and each teacher was to teach in a specially equipped room. From the foregoing it would appear that a teacher dealt on a one-to-one basis with the pupils and that he would give assistance when asked. There is no indication that he would lecture or teach the entire class for any length of time. The specialist requirement would seem to indicate that the teacher was expected to diagnose and guide the pupil to the proper learning task in the academic subjects. The writer speculates with respect to the academic subjects only since that approach is of importance to the present study.

The Winnetka plan also dates back to 1919. This plan also divided the curriculum into two parts. The one part contained the basic subjects which each pupil had to master and the other part contained group and creative activities. In the latter areas the pupils had no specific standards to meet. Each part of the curriculum had a half-day devoted to it every day. The basic subjects were divided into unit lessons. The pupils worked these until they felt ready to take the mastery test (Anderson, 1966: 78). The approach to the basic subjects implies a form of programmed instruction, where the teacher would appear to act as a resource person. Anderson (1966) indicates that the teachers planned as a team in that they had a form of co-operative planning and program development. Again the writer is interested in the basic subject area of the curriculum and the teacher's role in that setting. It appears once again that the teacher in that

setting mainly deals on a one-to-one basis with the pupils. It is difficult to state what the teacher actually did. The setting, however, seems to imply a form of individualized learning.

In the foregoing two plans the role of the teacher would appear to be distinctly different from the normally accepted role. His efforts would appear to have been directed toward dealing with the individual pupil and would appear to be a role of directing and guiding the pupil. The setting definitely provided the opportunity to help each pupil with his individual problems and difficulties. The attempt was to try to provide for individual differences.

Presently several plans are in the process of being evaluated. Each of these plans has the aim to provide for individual differences. Among these are I.M.U. in Malmö, Sweden; Project Plan in Palo Alto, California; I.P.I. in Pittsburgh, Pennsylvania; and individually guided learning in Milwaukee, Wisconsin.

Before describing in more detail any of the plans mentioned in the previous paragraph, the writer would like to refer to an article, published by Bloom in 1968, in which he gave an outline of a possible model for individualized instruction. In this paper he implies that the teacher has to be able to form a diagnosis of the pupil's needs on the basis of what Bloom calls a "formative evaluation". The formative evaluation is based on diagnostic-progress tests. On the basis of these tests the pupil is told if he has reached mastery of a unit of work or not. If the pupil has not reached mastery the teacher has to prescribe the appropriate learning situation, the appropriate learning task, and the appropriate materials for the particular pupil.

Thus Bloom seems to suggest that the teacher perform the role of a diagnostician and prescriber. Bloom speculates that no learning technique or instructional method can be overlooked in a program of learning for mastery. The teacher role is implied to be a complex one, for in part Bloom states:

> We suspect that no specific learning material or process is indispensible. The presence of a great variety of instructional materials and procedures and specific suggestions as to which ones the student might use help the student recognize that if he cannot learn in one way, alternatives are available to him. (Bloom, 1968)

The task for the teacher becomes even more complex when Bloom suggests that at present there is no firm evidence on the relations between student characteristics and instructional materials and procedures. The role of the teacher in the setting Bloom suggests is one of dealing on a one-to-one basis with the pupil and that he has to diagnose and prescribe.

The developers of the IPI method have implied much the same role for the teacher as Bloom suggested. They say that the teacher's function is:

> To administer placement and pre-tests, diagnose needs of learner, prescription writing (review background information and test scores), analyze student progress, provide guidance, prescribe post-test, and determine mastery. (Scanlon and Bolvin, N.D.: 11)

Thus the main emphasis for the function of the teacher is on diagnosis and prescription with guidance. At this time the materials to be used have all been prepared and the teacher has to make the selection from among them as well as to determine if the pupil will have large group instruction, small group instruction, individual tutoring or if he will work independently on the programmed material.

The task in this setting has been simplified to some extent as compared to what Bloom suggested. The teacher has a finite number of provided alternatives to select from.

To gain a little more insight into what the teacher role in the IPI setting is, consider the following quote.

> One rarely finds a teacher lecturing in an IPI class. He is too busy carefully observing each child's progress, evaluating his diagnostic tests, writing prescriptions, and instructing individuals or small groups of pupils who need help. The teacher's role is what experienced teachers have been seeking for many years -- that of teaching, evaluating, and making important educational decisions. All individual pupil prescriptions are reviewed by the teacher every day since the majority of the prescriptions do not last more than one class period. (Deep, 1968)

For the IPI project a program of teacher training has been outlined. The aims outlined in this program do give some indication as to what the teacher is expected to be able to do in the IPI setting. The teacher is expected to be able to specify learning goals in terms of behavioral objectives. He must be able to assess a pupil's achievement of his learning goals. Then he must be able to diagnose the learner's characteristics as well as being able to plan long-term and short-term learning programs with pupils. Along with this he must be able to guide pupils in their learning tasks and be able to direct off-task pupil behavior. Finally he must be able to evaluate the total learner. Another requirement which is considered essential is the ability to work with his colleagues as a member of a team as well as being able to enhance the full development of a child's potential (Southworth, 1968).

The IPI program has been described and detailed reasonably well in the literature. At present the evaluation of the program is

continuing.

Another program for individualized instruction has been developed by the American Institutes for Research, Palo Alto, California. This project is known as Project PLAN. The name of the project in a sense gives the main idea about the program; PLAN stands for Program for Learning According to Needs. The basic idea for this plan is to use existing commercially prepared materials rather than specifically prepared materials. A computer is used to assist the teachers in the evaluation of a pupil's progress. The program is just now ready for large scale testing (Phi Delta Kappan, April, 1970: 456). "Project PLAN is an attempt to mobilize the tools, the technology, and the procedures of modern education behind the teacher in a systematic and comprehensive attempt to organize instruction according to the needs of the individual student" (Mager, 1967). It is difficult to state precisely what the teacher's role is intended to be, but their literature seems to imply that the teacher again has a role of diagnosing, guiding, and evaluating. The teacher does not prescribe. The last conclusion has been drawn from the statement:

> The student is provided the option of choosing among relevant instructional procedures the one or ones judged <u>by</u> him to be most suitable <u>for</u> him. (Mager, 1967)

The above statement does imply, however, that the pupil has been supplied, by the teacher, with a number of alternatives suitable for him. The teacher seems to have therefore a definite guidance role. To be effective in this role the teacher must be able to evaluate and diagnose accurately.

At the University of Wisconsin a program for individually

guided learning has been developed. This program has an added feature in that it has also developed a new concept of administration and organization of the school staff. It is hoped that the new administrative organization will facilitate the program of individually guided learning. The new organization of the staff gave rise to the multiunit school. There is at least one unit per age level under the leadership of a unit leader. The unit leaders together with the principal form the administrative unit of the school. The emphasis in this program is upon guidance. The teachers are expected to guide carefully the pupils in their learning experiences (Klausmeier, 1969). The teacher role in this setting is one of guiding which in turn implies that the teacher must be able to evaluate and diagnose.

In Malmö, Sweden, yet another project has been developed for the purpose of individualizing instruction. The IMU project will have completed its first major testing period in 1970. In order to be able to state what the teacher role may be in this program it will be necessary to describe the program. Their literature does not make any specific reference to the expected role of the teacher. The approach used for the IMU can be shown best by a diagram, which shows how a module of work is covered.

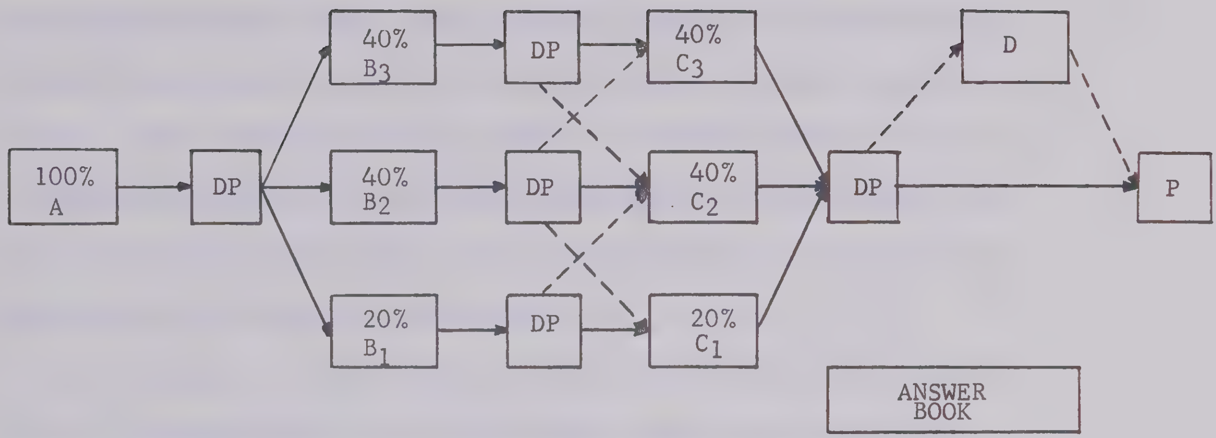


FIGURE I

SCHEMA SHOWING IMU MODULE COVERAGE

Three years of work has been divided into nine modules. Each module is covered as shown in the diagram. Hence all pupils cover the A component together, then on the basis of a post test the pupils are divided into three streams and put in the B component as shown, where B_3 is for the low achiever and B_1 is for the high achiever. At the end of the B component post tests are given and a reassignment of levels is made before a pupil goes on to component C. Finally after the post test of component C further time is made available for enrichment in component D. Components A, B, and C are expected to take about one month each to cover, so that the teacher has to assess his pupils about ten times per year. For each component there is a separate booklet and answer key. Within each component there are diagnostic exercises for the pupil to score himself, on the basis of which he proceeds through the work (Öreberg, 1968).

One aspect of the teacher role appears to be that of a resource person, for no mention has been made of large group instruction or small group instruction. Another aspect appears to be the task of evaluating, diagnosing, and assigning of pupils to their proper learning level. These aspects would imply that the teacher should be able to diagnose and guide pupils in their learning activities. However as was stated before, the stated role of the teacher has been based on speculation on the part of the writer.

All of the major studies mentioned up to this point give very little information about what the teacher does in the particular setting. It is even difficult at times to determine what his role in the broadest sense is supposed to be. Even IPI reports are still in general terms. What the teacher does specifically and how much time he devotes to it is not reported. Of course, it has to be realized that all of these programs are still considered to be in the experimental stage. Perhaps it cannot be expected as yet to receive any information in published form which deals with specifics.

Besides these major studies, studies on a smaller scale are being attempted. Some are attempted by a single teacher in a single class. Many of these attempts will never be known, for these are not being documented and reported. Nevertheless many of these could be classified as attempts at individualization of instruction. Wade (1968) stated that any attempt which satisfies the condition stated by Glaser does constitute a form of individualized instruction:

> Individualization of instruction is the adaptation of instructional procedures to the requirements of the individual learner. (Glaser, 1965)

An example of a single teacher attempting a program which satisfies the above condition was reported in the SCAT Bulletin (published by the Science Council of Alberta Teachers Association) of June, 1969, where Lorincz outlined a program of self-study in Chemistry 30X. Actually some writers encourage teachers to individualize instruction on their own. Hunter (1970) published an article outlining how to adapt one's teaching style to a form of individualized instruction. The fact remains however that none of the literature mentioned described in detail what a teacher does in an individualized setting.

Mortlock (1969) reported a style of teaching which attempted to cater to individual differences. His instructional model had two phases for each topic studied. All the objectives were stated by the teacher in behavioral terms. During phase I Mortlock taught all the children at an intermediate level. During this phase the teaching style did not differ appreciably from the regular style of teaching. At the end of phase I the pupils wrote a post-test on the basis of which the teacher placed pupils in one of three streams for phase II. In phase II the program took the form of independent study with the teacher as a resource person ready to give individual help. In outline, this program is actually similar in format to components A and B of the IMU program, except that the teacher taught the first phase. In the second phase the teacher worked mainly with individual pupils and small groups.

The individualized instruction reported in this thesis was based in format upon the study by Mortlock. The phase I, however, was also covered in an independent study form, but all the students worked the material in phase I at the intermediate level. For the

description of the study see the section "The Experimental Setting" elsewhere in this thesis.

In the next section the writer will report briefly on the literature related to the classroom observation techniques.

<u>Literature Related to Classroom Observations</u>

For many years forms of observation techniques have been used. It was apparently believed that an expert in education could determine by observation if a teacher was an effective teacher and this belief is still held. Superintendents, inspectors and faculty consultants visit the classroom for the purpose of evaluating a teacher by observation. The process is very subjective and difficult to use in any rigorous study. More objectivity is desired. Several attempts have been made to make the evaluations more reliable and objective.

The earliest attempt at using a recording technique for the purpose of evaluating a teacher's effectiveness appeared to have been made by Horn (Medley and Mitzel, 1964: 254). Horn was a school inspector and hence vitally interested in the performance of the teachers. He believed that through the recording of the verbal interactions between pupils and teachers one could determine the effectiveness of a teacher. Horn took a seating plan and placed a circle in the appropriate square for each recitation a pupil made. He thus attempted to record to some extent the verbal interactions between pupils and teacher.

Puckett in 1928 tried to improve upon the primitive scheme used by Horn (Medley and Mitzel, 1964: 254). He still used the idea of a seating plan, but he added different symbols to the circles to

indicate what type of response a pupil gave or if he failed to give a response. He thus obtained more detailed information than Horn did. Next, Wrightstone in 1934 developed a list classifying verbal responses made by the teachers and pupils. Each class of verbal behaviors was given a code number (Medley and Mitzel, 1964). Wrightstone still used the seating plan but recorded a code number each time the teacher spoke to a pupil and the pupil responded or failed to respond. This approach was an improvement over Puckett, but at the same time it had become much more complicated in that the observer had to memorize a set of categories and had to make finer judgments regarding the categories to which a statement belonged. Since the above attempts many more sophisticated tools have been developed (Medley and Mitzel, 1962).

Among the techniques developed in recent times are the ones by Flanders, Proctor and Wright, Hughes and B. O. Smith. Flanders developed a set of categories which were designed to group the verbal behaviors of the teacher and the pupils in nine different categories. Seven of these categories dealt with teacher talk. These seven categories were subdivided into two groups: four referring to indirect influence and three referring to direct influence. Two categories were to analyze student talk. Finally one category referred to silence or confusion. The categories were not simple statements, but were actually quite complex. For instance, category 3 reads:

> Accepts or Uses Ideas of Student: clarifying, building, or developing ideas suggested by a student. As teacher brings more of his own ideas into play, shift to category 5.

Category 5 reads:

> Lecturing: giving facts or opinions about content or procedure; expressing his own ideas, asking rhetorical questions. (Flanders, 1961)

From the above example it becomes clear that the observer has to make some rather complex decisions. On Flanders' scales the observer oberves for 3 seconds and then makes notations during the following 3 seconds. Hence quite a complete record could be obtained with respect to the verbal behavior.

Bales and Gerbrands (1948) designed a machine which would provide them with a continuous running tape alongside their card containing the categories. The paper tape passed at a constant speed by the card at all times. Thus they were able to determine the exact lapse time and the duration of any verbal behavior recorded. Their categories were strictly verbal interaction oriented and based on group dynamics.

Another approach was used by Hughes. She made an exact transcript of what happened in the classroom. For this purpose Hughes used a team of observers to collect the data. The approach was very elaborate and exacting, but more than one person was required to make the observations. Once the observations were completed and the transcripts were made various analyses could be made.

B. O. Smith used tape recordings in his study, and then transcribed from the tape what occurred. Again various analyses could be made once the transcripts were made.

Wright and Proctor developed an observation instrument to be used in mathematics classes only. A training manual to be used for this instrument has been published. The categories were first in relation to content, process, and attitude. Each of the areas were subdivided. For instance under content the six subheadings are:

C_1 structure; C_2 deductive; C_3 mathematical; C_4 other; C_5 techniques; and C_6 inductive. So every statement is analyzed with respect to content, process, and attitude. Then the observer has also to show who made the comment the teacher or the pupil. If the statement is made by a pupil the observer has to show if it is a boy or a girl. Finally the observer has to make an analysis about the class. He has to determine the rigor and the participation. The process is complex and the observer has to work quickly. He observes for 15 seconds and makes notations during the next 15 seconds. The observer also has to be an expert in mathematics and has to be very familiar with the particular subject he is observing. The latter becomes evident if the categories under process are considered. These are: P_1 analyzing; P_2 synthesizing; P_3 specializing; P_4 generalizing; and P_5 relevant. The observer has to memorize the code for each category. The authors of the instrument report that at least ten daily observations per teacher are needed in order to gain validity.

Cornell, Lindvall and Saupe in 1952 developed an instrument which was not entirely dependent upon verbal interaction. This instrument and the categories developed by Withall in 1949 were used by Medley and Mitzel to develop an observation instrument called the OScAR (Observation Schedule and Record). Later that year, 1958, they revised it to make it more reliable. The categories in this instrument are all stated in behavioral terms and refer to a single behavior. For instance they use statements like: "the teacher lectures" or "the teacher asks a question, the pupil answers". The observer does not have to analyze the statement or action as to which category it

belongs. The instrument was designed in such a way that the observer had to make a minimum of decisions and it was easy to score (Medley and Mitzel, 1964).

The instrument contained a large number of categories as a consequence, but the observers had to be less highly trained. The authors of the OScAR claimed that an instrument will do its job as long as it records the type of behavior that was intended to be examined. They define an observational technique as follows:

> An observational technique which can be used to measure classroom behavior is one in which an observer records relevant aspects of classroom behaviors as (or within a negligible time limit after) they occur, with a minimum of quantification intervening between the observation of a behavior and the recording of it. Typically, behaviors are recorded in the form of tallies, checks, or other marks which code them into predefined categories and yield information about which behaviors occurred, or how often they occurred during the period of observation. (Medley and Mitzel, 1964)

Shaver showed that an instrument does not have to be extremely elaborate. He predetermined a number of behaviors he would like to compare in two different settings and used a very simple scoring sheet to record the frequencies of occurrence. His scoring technique was similar to that used by Puckett. In a sense what Shaver did had been stated earlier by Medley and Mitzel.

After reviewing a number of techniques the writer had to decide which technique could be adapted to his purposes. The ideas used in many were strictly related to verbal interaction and as such were out of the question from the start, because the writer could expect to have extended periods of silence occurring in the classrooms which he was to observe. This eliminated scales by Horn, Puckett, Wrightstone, and Bales and Gerbrands. The techniques used by Hughes

and B. O. Smith were also eliminated. The technique by Hughes was too elaborate and the writer did not have the resources to do anything as elaborate as Hughes did. The technique used by Smith is intriguing except a taperecorder can only reproduce audible behaviors and it is thus limited to a measure of verbal interaction only.

The instrument by Wright and Proctor appeared to be a very fine instrument. Yet it had two shortcomings as far as the writer was concerned. First, there were too many observations necessary per teacher and second, there was a major shortcoming as reported by one of the authors. She stated the following:

> The behaviors classified with this instrument are those of verbal interaction or its attendent demonstration. Thus the description of the lesson period yielded by the instrument is limited to the amount of verbal class interaction occurring. In classrooms where long periods of supervised study are regularly used, this limitation may be so great that a picture of the in-lesson period by this instrument would be impossible. (Wright, 1959)

Another examination of the instruments considered to this point shows that there are basically two types of instruments. The one group of instruments is designed to get a qualitative measurement. In other words the observer evaluates the behaviors as he records them. Instruments in this category are those designed by Flanders, Wright and those which were based on either of these instruments. The other group is those that use a quantitative approach. The basic concern is how often did it occur and not what category should it be placed in after the behavior has been observed. The OScAR technique belongs to this group. In part Medley has this to say about it:

> Our approach differs from that used by most other groups interested in the teaching process in that it has been basically a measurement approach. Our goal has been to develop procedures for obtaining objective quantitative descriptions of teacher behavior, in terms of a minimum number of dimensions, on the basis of direct observation. (Medley, 1966)

The latter technique would be useful for the writer in his study. The writer needed an instrument that would record verbal as well as nonverbal behavior, preferably in a quantitative manner as it was not the purpose of the study to evaluate the teachers. Almost all of the observation schedules were designed for the purpose of evaluating teachers. However, none of the techniques described have as yet attained that objective fully. Morsch and Wilder found in 1954 that the results on teacher effectiveness for instance did not correlate with pupil achievement. They concluded that it was not yet possible to assess effectiveness of a teacher by observation alone. Also, judging by a recent article in the Kappan it would appear that as yet this goal has not been achieved (Rosenshine, 1970). Thus the instruments noted could all have been used by the writer if they had not had the shortcomings cited for each.

Finally, it is important to see what the literature reports about the desirability of classroom observation. Noting what some of the prominent researchers have to say might be of value. B. O. Smith states:

> If very little is known about a phenomenon, the way to begin an investigation of it is to observe and analyze the phenomenon itself. (Smith, 1959)

Wispe (1953), Morsch and Wilder (1954) and others suggest that classroom observation is a potentially fruitful source of data in any research involving behavior in the classroom (Shaver, 1961).

On this point Flanders states:

> It is clear, from our research that many factors affect patterns of teacher statements. Teachers of different grade levels and of different subject matter will produce radically different patterns of verbal behavior. (Flanders, 1961, p. 179)

The writer found that Medley and Mitzel also consider classroom observations to be important. They say, in part:

> It seems safe to say that almost any research on teaching and learning behavior can benefit by the use of direct observations of the behaviors, and that in many instances such observations are of crucial importance. (Medley and Mitzel, 1954)

From the above statements it appears that the notion to use direct classroom observations is worthwhile in conjunction with the present study, because the phenomenon is basically unknown and the teacher behaviors are of the utmost importance in this program.

The only question that remains is "Can observation schedules be used in the comparison of two teaching styles?" This question was answered by Nelson (1966) when she used Flanders' categories to compare two teaching styles in Social Studies.

Summary

From the survey of the literature it becomes clear that most studies only vaguely imply what a teacher is expected to do and some describe in reasonable detail what is expected of them. Specific details about what a teacher did and how much time the teacher spent on certain tasks are not reported. Mortlock and IPI reported some detail about the general classroom tasks a teacher performed, but specific details were not mentioned. Many of the researchers have perhaps not given full attention to this aspect. The teacher is perhaps taken too much for granted. For instance at a recent conference on individualized

instruction, held in Banff, Alberta, Bolvin was asked how much time and effort was involved for the teacher using the IPI method. The reply by Bolvin was that the IPI project demanded teachers who were dedicated and who were willing to put in long hours of hard work. He was not able to specify how many hours per week were required (Banff Conference, 1969).

It appears that it is worthwhile to describe what a teacher does in an individualized setting for mathematics and how much time he spends doing in-class as well as out-of-class activities. When two programs are being compared the criterion used to determine which one is the better should involve an assessment of change in what the teacher must do (Lindvall, 1966). Lindvall goes on to say that too frequently evaluation efforts are centered on measures of achievement only when the real purpose of the innovation is something quite different.

It appears also that an observation technique based on the OScAR technique will satisfy the need for the purpose of the present study. Medley (1966) suggests that his instrument might be very useful in projects whose purpose it is to implement educational change. The reason the writer has selected the approach by Medley and Mitzel is that the instrument is behaviorally oriented and that the observer will have a minimum of classifying to do when he is recording the behaviors he sees. Having arrived at the above decision the next step was the development of the instruments. In the following chapter the design and the preparation of the instruments will be discussed.

CHAPTER III

DESIGN AND PREPARATION OF INSTRUMENTS

Introduction

To describe a phenomenon it is necessary to observe it and collect systematically as much data about the phenomenon as possible. To collect data systematically instruments have to be designed. Instruments cannot be designed unless the aspects about the phenomenon to be recorded have been determined. The writer found it necessary to formulate a set of general questions about the phenomenon in order to be able to determine what techniques and instruments to use in the gathering of the data necessary for the answering of the questions. The questions would also assist the writer in the description of the phenomenon. The following general questions were formulated:

1. What does a teacher do in the classroom?
2. How much time does he spend on the various tasks related to his teaching?
3. How does he plan?
4. What are his reactions to the program?

The writer decided to do actual classroom observation for the investigation of the first question. In order to be able to record what was observed, the writer developed an Observation Schedule, which was patterned after the OScAR technique by Medley and Mitzel. For the investigation of the second question the writer developed a weekly log sheet. He asked each teacher to keep a log sheet for the

duration of one week at three different times during the experiment. Finally, to investigate questions three and four the writer thought it best to use a questionnaire, which he developed for this purpose. Each of the instruments will be discussed briefly in the following sections.

The Observation Schedule

After considering a number of instruments (see Chapter II) the writer selected the approach used by Medley and Mitzel in their instrument called the OScAR. Modifications were made because the writer had different objectives in mind than Medley and Mitzel. They were using the instrument to investigate the change in behavior of student teachers. The writer wanted to be able to record a phenomenon observed. Specifically the writer set out to describe what the teachers did with respect to some general mathematics classroom procedures. To be able to gain a fuller picture of the situation pupil behaviors were recorded as well. In a classroom the teacher's behavior and the pupil's behavior are often very closely linked and the two behaviors generally seem to complement each other. The writer set out to investigate the following classroom procedures: Checking of assignments and homework; formal teaching; directed practise; testing; seat work; and laboratory work. Linked with these are managerial tasks, grouping, and classroom climate. Finally a section on the affective teacher talk and actions was included. Under each of the above mentioned topics the writer listed possible behaviors on the part of the teacher or pupils. As an example, consider the section on formal teaching. The entire instrument appears on page 34 and in Appendix B.

2.0 Formal Teaching

2.1 teacher lectures
2.2 teacher uses board
2.3 teacher uses map, chart, etc.
2.4 teacher uses slide film, etc.
2.5 teacher uses audio aid
2.6 teacher uses object
2.7 teacher uses special teaching aid
2.8 teacher uses no materials
2.9 teacher questions, pupil answers
2.10 teacher answers pupil's question
2.11 teacher ignores pupil's question
2.12 teacher refers pupil's question
2.13 teacher leads discussion
2.14 teacher reviews previous work
2.15 teacher uses hand out
2.16 teacher reteaches
2.17 pupil demonstrates at the board
2.18 pupil leads class discussion
2.19 pupil works example at seat
2.20 pupil works example at the board

FIGURE II

SAMPLE OF A SECTION SHOWING BEHAVIORS LISTED

Thus under each topic the writer listed a set of behaviors that could possibly be observed. On the following pages an example of the OScAR and the schedule used by the writer will be shown, so that the reader can observe the similarities and the differences. First the OScAR will be shown and then the schedule developed by the writer will be given. The OScAR was printed on both sides of a cardboard. The front of the cardboard will be shown on page 31 and page 32; and the reverse will appear on page 33. The reader will then see the observation schedule developed by the writer on the pages following. The OScAR shown is as adapted from Medley and Mitzel (Gage, pp. 278-280).

OScAR

Tm No. p.

Tot		I	III	V
	A0 (TP-PT)	X	X	X
	A1 t wks w ind p			
	A2 t wks w sm gp			
	A3 t qu, p ans			
	A4 t ans p qu			
	A5 t ign p qu			
	A6 t lds sng, ex, gm			
	B0 (TP)	X	X	X
	B1 t lectrs			
	B2 t rds, tls sty			
	B3 t tks to cls			
	B4 t illus at bd			
	B5 t illus at mp, cht			
	B6 t dmnstrs			
	B7 t shws fm, sld, plys rcd			
	B8 t pss ppr, bks			
	C0 (T)	X	X	X
	C1 t wrks at dsk			
	C2 t clns, decrts rm			
	C3 t wrts on, dcrts bd			
	C4 t tks to vstr			
	C5 t lvs, entrs rm			
	Check			

	I	III	V	Tot
D0 (P)	X	X	X	
D1 p rds, stdys at st				
D2 p wrts, mnps at st				
D3 ppnts,cts,drws,etc.				
D4 p wks at bd				
D5 p dcrts rm, bd				
D6 p clns rm, bd				
D7 p rsts, has snk				
D8 p lvs, entrs rm				
D9 p pts hnds on hd, etc.				
E0 (PP)	X	X	X	
E1 p tks to gp				
E2 p rcts				
E3 p rpts, gvs prpd tk				
E4 p rds ald				
E5 p dmnstrs, illus				
E6 p gvs skt, ply				
E7 p sngs, pl instr				
E8 p plys gm				
E9 p interps				
E10 p lds cls				
F0 (PM)				
F1 p ign t qu				
F2 p scfls, fts				
F3 p wsprs				
F4 p lghs				
F5 p pss ppr,bks,mlk				
F6 p tks to vstr				
Check				

FIGURE III

PART OF FRONT OF OScAR 2a AS ADAPTED FROM MEDLEY AND MITZEL

OScAR (Cont'd.)

Tot	I	III	IV	t (Mtls) p	I	III	V	Tot
				L1 Blbd				
				L2 Mp, Cht, Pctr				
				L3 Sld, Flm, etc.				
				M Audio Aid				
				N5 Obj				
				N6 Spec Tchg Aid				
				O No Mtls				
				P1 Txt, Wkbk				
				P2 Supl Rdg Mtr				
				Q Wrtg				
				R Hcft, Art				
				Check				

Tot	I	III	V	Soc (Gpg) Adm	I	III	V	Tot
				G1 at 1st ½ cl in gp wt				
				G2 at 1st ½ cl in gp w/ot				
				G3 4 p to ½ cl in gp wt				
				G4 4 p to ½ cl in gp w/ot				
				G5 2-3 p in gp wt				
				G6 2-3 p in gp w/ot				
				G7 p as ind				
				Check				

(Sns)	I	III	V	Tot
S2 t mvs frly				
S3 p mvs frly				
S5 t cls p dr, etc				
S6 t shws afct f p				
S7 p shws afct f t				
S8 p shws ho t t				
S9 p shws ho t p				
S10 t uses srcsm				
S11 t yls				
Check				

FIGURE III CONTINUED

OScAR (Cont'd.)

	II	IV	VI	Tot
K1				
K2				
K3				
K4				
K5				
K6				
K7				
K8				
Chk				

H. Rcp

I1 TI	I2 PD	I3 Tr

J1 DO	J2 NL	J3 RP

Tot	II	IV	VI	(Sbj)	I	III	V	Tot
				T1 Rdg				
				T2 Math				
				T3 Lang Arts				
				T4 Soc St				
				T5 Science				
				T6 Recreation				
				T7 Art, Crafts				
				T8 Music				
				T9 Soc Process				
				T10 Test				
				Check				

FIGURE IV

BACK OF OScAR 2a AS ADAPTED FROM MEDLEY AND MITZEL

OBSERVATION SCHEDULE

Tot	1.0 CHK ASSM	I	III	V	4.0 MANGRL TSKS	I	III	V	Tot
	1.1 t wks ex at bd				4.1 t tks attnd				
	1.2 t rds hw answ				4.2 t clns bd				
	1.3 t asks for answ				4.3 t discpl				
	1.4 t chks ind hw				4.4 t mrks assm				
	1.5 t cmmnts on hw				4.5 t pss out mat				
	1.6 p wrks ex at bd				4.6 t wrks at dsk				
	1.7 p hnds in hw				4.7 t ans dr/phn				
	2.0 FORM TCH				4.8 t lvs/ents rm				
	2.1 t lects				4.9 t wrts on bd				
	2.2 t uses bd				4.10 t mks assm				
	2.3 t uses mp, cht, etc.				4.11 t mks annomnt				
	2.4 t uses sld, flm, etc.				4.12 p clns bd				
	2.5 t uses aud aid				4.13 p pss out mat				
	2.6 t uses obj				4.14 p clns rm				
	2.7 t uses sp t aid				4.15 p ans dr/phn				
	2.8 t uses no mat				4.16 t tlls p to hlp p				
	2.9 t qu, p ans				5.0 TESTING				
	2.10 t ans p qu				5.1 t supv tst				
	2.11 t ign p qu				5.2 t supv quz				
	2.12 t ref p qu				5.3 p wrts tst/quz				
	2.13 t lds disc				5.4 cls maks/tks up tst				
	2.14 t rvws prev wrk				5.5 t ign p qu				
	2.15 t uses hnd out				5.6 t ans p qu				
	2.16 t retchs				6.0 ST WRK				
	2.17 p dem at bd				6.1 t hlps ind				
	2.18 p lds clss disc				6.2 t wks w sm gp				
	2.19 p wrks ex at st				6.3 t urg p to wrk				
	2.20 p wrks ex at bd				6.4 p st, rds at st				
	3.0 DIR PRACT				6.5 p wrts, mnps at st				
	3.1 t expl				6.6 p clrs, drws, cts, etc.				
	3.2 t chks ex wrkd				6.7 p asks qu, t hlps				
	3.3 t retchs idea				6.8 p uses hnd out				
	3.4 t hlps ind				6.9 p uses txt				
	3.5 t wrks prbl				6.10 p uses wrkbk				
	3.6 p wrks ex at st				6.11 p uses auth rdg				
	3.7 p dem at bd				6.12 p hlps p				
	3.8 p hlps p				6.13 t expls to cls				
	3.9 p tlks to sm grp				6.14 t chks prbl wrkd				
	3.10 t qu, p ans				7.0 LAB WRK				
SUBJ. OF LSSN TGHT:					7.1 t drcts p to lab				
NO. PUPILS IN CLASS:					7.2 p uses pzz, gms				
TCHR. OBS.:					7.3 p uses suppl rd				
DATE:					7.4 p uses calc, etc.				

FIGURE V

FIRST PAGE OF THE OBSERVATION SCHEDULE

The writer had two sheets side by side in a manilla folder. The first sheet has been shown on the previous page. On the next page the second sheet of the schedule will be shown.

				Soc Adm				
Tot	I	III	V	8.0 GROUPG	I	III	V	TOT
				8.1 at 1st ½ cl in gp w t				
				8.2 at 1st ½ cl in gp w/o t				
				8.3 4 p to ½ cl in gp w t				
				8.4 4 p to ½ cl in gp w/o t				
				8.5 2-3 p in gp w t				
				8.6 2-3 p in gp w/o t				
				8.7 p as ind				

9.0 CLSRM CLIM	I	III	V	Tot
9.1 t mvs frly				
9.2 t discpl p dir				
9.3 t shws afct f p				
9.4 t shws host t p				
9.5 t uses sarcsm				
9.6 t yls				
9.7 p mvs frly				
9.8 p shws afct f t				
9.9 p shws host t t				
9.10 p shws host t p				
9.11 p ign t qu				
9.12 p scfls, frts				
9.13 p wsps				
9.14 p lghs				
9.15 p lvs/entrs rm				
9.16 p l st				
9.17 nse and tme wstng				
9.18 p works at bd				

REMARKS:

OBSERVATION OF AFFECTIVE TEACHER TALK AND ACTIONS 10.0	II		IV		VI		Tot	
	IND	GP	IND	GP	IND	GP	IND	GP
10.1 t apprvs, aft gest, lk								
10.2 mks p supportv stmnt								
10.3 mks prblm strctg stmnt								
10.4 mks misc stmnt								
10.5 mks drtve stmnt								
10.6 mks reprvng stmnt								
10.7 reprvng host gest, lk								
10.8 t assrts authrty								

FIGURE VI

SECOND PAGE OF OBSERVATION SCHEDULE

The reader will observe that the grouping section in the OScAR and section 8.0 in the Observation Schedule are identical. The K section in the OScAR and section 10.0 in the Observation Schedule are also identical except the writer inserted the behaviors. The T section of the OScAR was omitted in the Observation Schedule, for the writer was observing mathematics classes only. The resemblance between the OScAR and the Observation Schedule ends here except for the abbreviations used. The main reason for altering the remaining format was that the writer was interested in specific behaviors at specific times, whereas the authors of the OScAR were interested in general behaviors and interactions. They would group the various behaviors into keys to gain a description of classroom climate and so on. To form the keys other criteria had to be known and a statistical analysis for interaction of various behaviors had to be performed. The writer felt that for the purpose of recording of classroom activities the OScAR would not give him enough information. For instance the OScAR had no item showing that a pupil helped another pupil. Another reason for changing can best be illustrated by the following item: "teacher answers pupil's question", which appears only once on the OScAR. From the OScAR one could not tell if this occurred in a formal teaching situation, testing situation, or in the situation where pupils were working at their desks. The writer wanted this information and hence the modifications were made. After the writer had constructed his Observation Schedule he had it examined by a panel of experts at the Faculty of Education, University of Alberta. Suggestions from the panel were incorporated and alterations were made accordingly. The basic design of the instrument was not questioned or changed. The

suggestions for change were all behaviors which the writer had not included under a certain topic. For instance under the topic of "Checking Assignments" it was suggested to include the following: teacher comments on homework.

Next the writer took the schedule and tested it in a classroom situation to see if it did suit the purpose for which it was designed. A small number of observations were made in two grade six mathematics classrooms where the IPI method was used. The reason for selecting these classes was that the IPI method was the closest in resemblance to the method used for the experiment. A co-observer was used in this test to see if observer agreement could be obtained with the instrument. Results of this test will be reported later in a section on the Pilot Study. After the observations were made the results were discussed with the teachers to see if they thought that the results were in agreement with what they believed happened in the classroom. Generally speaking, the teachers felt that the results were quite accurate. Some of the details that were recorded the teachers could not recall or were not aware of them, such as gestures or facial expressions as recorded in section 10.

In the next section scoring of the Observation Schedule will be discussed.

I. Scoring of the Observation Schedule

The scoring technique used is the same as that used for the OScAR. The instrument actually consists of two basic parts. Part I comprises sections 1 to 9 inclusive. Part II is section 10. The observer checks in alternate five minute periods as many items as he

can in Part I. For instance in the first five minute period he might check in column I items 1.2, 1.5, 4.1, 4.3, 4.15, 8.1 Adm., 9.1, 9.3, 9.8, 9.11, 9.13, 9.14, 9.16, and 9.17. (Return to pages 34, 36 to consult the schedule.) It does not matter how often the same behavior recurs in a five minute period; it is checked only once. In part II however each category is checked every time it occurs during a five minute period. A true frequency is obtained in section 10, whereas the other nine sections only give a sample of what occurred. Basically the Observation Schedule is a sampling technique. An observer can thus gain a reasonable picture of what is happening in a classroom, providing the sample he took is large enough.

Returning now to the scoring of the instrument. An observer checks as many categories as he observes in the first five minutes in part I. He places a check mark in column I for each category observed. For the second five minute period he turns to section 10 and places a check mark in column II for the appropriate category every time it occurs. Then for the third five minute period he returns to part I and checks all categories observed during that period in the column marked III. Next for five minutes he enters checks in the column marked IV, and so on. As an example section 4.0 and section 10.0 will be shown checked. Also the totals are shown. In part I the total indicates actually the number of five minute observation periods during which the behavior was observed. A quick glance at the sample will confirm this to the reader.

4.0 MANGRL TSKS	I	III	V	Tot
4.1 t tks attnd	✓			/
4.2 t clns bd		✓	✓	2
4.3 t discpl	✓		✓	2
4.4 t mrks assm				
4.5 t pss out mat		✓		/
4.6 t wrks at dsk			✓	/
4.7 t ans dr/phn	✓			/
4.8 t lvs/ent rm				
4.9 t wrts on bd				
4.10 t mks assm		✓		/
4.11 t mks annomnt	✓			/
4.12 p clns bd				
4.13 p pss mat				
4.14 p clns rm				
4.15 p ans dr/phn				
4.16 t tlls p to hlp p			✓	/

FIGURE VII

A SAMPLE TO ILLUSTRATE SCORING OF FIRST NINE SECTIONS

OF OBSERVATION SCHEDULE

10.0	II		IV		VI		Tot	
	IND	GP	IND	GP	IND	GP	IND	GP
10.1 apprvng, afct gest, lk	✓✓	✓✓✓✓✓		✓✓✓	✓✓✓		5	8
10.2 mks p supportv stmnt	✓✓✓✓✓✓✓✓✓			✓	✓✓✓✓✓		14	1
10.3 mks prblm strctg stmnt		✓✓✓✓✓✓✓✓✓✓✓		✓✓✓✓✓✓✓✓✓✓✓✓✓✓✓	✓✓✓✓✓✓	✓✓✓✓✓✓✓✓	6	34
10.4 mks misc stmnt	✓✓✓✓✓✓✓✓✓	✓✓✓✓✓✓✓		✓✓✓✓✓✓✓✓✓✓✓✓✓	✓✓✓✓✓✓✓✓✓✓	✓✓✓✓✓✓✓✓✓	19	29
10.5 mks drtve stmnt	✓✓✓✓✓	✓✓✓✓✓✓✓✓✓✓✓✓✓		✓✓✓✓✓✓✓✓✓	✓✓✓	✓✓✓✓✓✓	8	28
10.6 mks reprvng stmnt		✓✓✓✓✓		✓✓✓	✓✓		2	8
10.7 reprvng, host gest, lk	✓✓	✓✓✓		✓	✓✓	✓	4	5
10.8 t asserts authrty		✓✓✓✓✓✓	✓	✓✓		✓	1	9

FIGURE VIII

A SAMPLE ILLUSTRATING THE SCORING OF SECTION 10

OF THE OBSERVATION SCHEDULE

The writer gave a hypothetical example above for section 10.0, however the distributions shown are similar to a regular classroom lesson.

II. Results of the Pilot Study

As was earlier stated, the writer and a helper observed two teachers in the IPI setting simultaneously to see if observer agreement could be obtained. After the first two observations some modifications were made in the instrument, i.e. 4.16 and 9.18 were added, so that observations could be recorded for which no item had been allowed. The tables that follow show the results based on the simultaneous observations. The totals for each behavior marked during each observation were put in a vector, formed according to the appearance of the behaviors on the schedule. Thus for each teacher and each observer and each classroom visit a vector of the totals was obtained. The vectors thus obtained by one observer were correlated with those of

the other observer. As can be seen from Table I the rank-correlations were low. (See page 44.)

The results shown in Table II were obtained after the instrument was modified as was mentioned previously. Much higher rank-correlations were found. Another factor that might have added to the increase in correlations is that the practise gained in the first set of observations might have influenced the results on the second set, for both observers had no previous experience in the use of the instrument. The significance here, however, is that the observers did record more frequently similar behaviors and thus made similar observations.

The third table shows the correlations obtained after the total scores on each item of the two observations for each teacher, by each observer, were totalled. The implication of this result is that on any given observation one observer might miss what another observer sees, but by combining a set of observations the tendency is that both observers observe the same items.

An interesting result was obtained when a t-test was run on the combination of the two observations. The result of this test is shown in Table IV. This shows that each observer checked the same number of categories for there are no significant differences in the mean number of their categories checked. This test does not imply that the two observers had observed the same behaviors, but that they marked the same number of categories. The correlations indicated, however, their tendency to observe the same categories during a series of observations. Thus it appears that the instrument makes it possible for two observers to record the same behaviors simultaneously and that

a series of observations will give a more reliable and complete picture than does a single observation. This finding is in agreement with what Medley and Mitzel reported. Thus it appeared to be essential for the writer to make as many observations per teacher as possible.

TABLE I

CORRELATIONS FOR OBSERVATION I

teacher / part	A	B
I	0.23	0.24
II	0.17	0.34

TABLE II

CORRELATIONS FOR OBSERVATION II

teacher / part	A	B
I	0.78	0.52
II	0.58	0.61

TABLE III

CORRELATIONS FOR OBSERVATIONS I AND II COMBINED

teacher / part	A	B
I	0.82	0.69
II	0.70	0.70

TABLE IV

T-TEST FOR COMBINATION OF OBSERVATIONS I AND II

teacher / part	A	df	B	df	critical t for $\alpha = 0.25$
I	0.069	66	0.200	62	0.679
II	0.53	14	0.53	14	0.692

The Log Sheet

The log sheet was developed to answer the question: "How much time does a teacher spend on the various tasks related to his teaching?" It seemed important to the writer to know what was demanded of the teachers participating in the experiment in terms of time. Work load and working conditions play an important part in any type of job and are thus of importance to the teacher. The writer chose those areas which, from his own experience as a teacher, were the most time consuming. These tasks were: planning lessons; preparing aids; marking assignments; preparing tests; marking tests; record-keeping; departmental meetings; and group planning. In order that the teachers would cooperate in keeping these log sheets, the writer felt that the task of keeping them should be held to a minimum. Thus the writer proposed a set of time intervals for each task so that the teacher would place a check mark for each activity under the appropriate time interval and beside the proper day. Thus it became a matter of checking appropriate squares on the grid of the sheet. The log sheet can be found in Appendix C. A glance at the log sheet will show the reader what the writer meant by the above statements. Every effort was made to keep the information on one sheet so that the amount of paper work for the teacher would be kept to a minimum.

Since all the teachers but one in the experimental program taught two experimental classes, the writer thought it appropriate to ask the teachers how they divided their class time among the various tasks in each class. The division of class time was difficult to record while the observer was scoring his observation schedule. Indirectly

some measure of division could be obtained from the schedule, but it would be a very rough approximation. The writer also felt that it would be of importance to see how the teachers viewed the use of class time and how much time they devoted to the various tasks in the classroom. Note that the categories asked for are strictly associated to the teaching of a group of pupils.

The writer prepared a log sheet based on the above stated objectives of preparing an easy to score single sheet and to gain information about out-of-class work load and in-class time allottments.

The same panel of experts that considered the Observation Schedule was asked to consider the proposed log sheet. A couple of minor changes were made in the wording of the headings. The word "for" was added to the first eight headings. The writer had varying time intervals for the different tasks, which the panel felt should be changed to equal intervals. The writer saw the merit in this, since it would make for easier comparison and analysis. The log sheet as adapted has been shown in Appendix C. The two teachers used in the pilot study for the Observation Schedule were also asked to comment on the schedule. They both felt that the areas covered by the sheet were appropriate and of greatest importance to the classroom teacher.

The log sheet would provide the writer with an insight into the work load the teacher faced if he did participate in the experimental program. Still, information as to how the teacher planned, prepared and cooperated with other teachers was not known, nor did the writer have any idea as to how the teachers felt about the program. To gain information in relation to these areas a questionnaire was prepared

by the writer which will be discussed next.

Questionnaire

The classroom behaviors of teachers were being recorded and also their workload in terms of time was recorded, but some aspects of importance remained unknown. It would be known for instance how much time the teacher spent planning and preparing, but still it would not be known how the teacher planned and how he prepared. Neither was it known how the teachers felt about the effect the program had on the different children, nor were the feelings the teachers had about the program known. The writer developed a questionnaire which would gather data related to the aforementioned unknowns and one which would be simple to answer and as short as possible. The writer once again felt that the teachers would give fair consideration to the questionnaire only if the length was kept to a minimum and the content directly related to the program. With this in mind the questions were formulated. First a section on personal data was included. This was done to see if there was any difference due to differences in experience or training. If no within-group differences occurred, then training and experience would not be of any statistical importance. Next questions were formulated with respect to the planning and preparation of a unit in grade seven mathematics. Following these a set of questions related to lesson planning was prepared. After that some general questions related to the handling of a class and the teaching approach were included. Next teacher expectations with respect to pupil achievement were examined. It was also important to know if a team approach to teaching was used or in what manner they taught. Much of a teacher's

attitude depends upon the amount of assistance he receives in the drudgery of a number of tasks, such as marking and record keeping. So questions about the teacher aide were included. Next a group of questions designed to gather opinions from the teachers as to how they saw the experimental program affecting their regular classroom teaching. The reactions to these questions would be of importance for inservice training and teacher training programs. Following these questions were questions relating to the feeling the teachers had about the program; how the program assisted them in helping various pupils; how the program affected different pupils; how the program affected teacher-pupil relationships and discipline. Finally two open-ended questions were included to give the teachers a chance to express their likes and dislikes. The latter reaction would be of importance for the revision of the program.

The writer originally had designed the questions in a multiple-choice form, but difficulties in answering those were pointed out by the same panel of experts that considered the previous instruments. After some discussion the Yes-No type of question seemed to be the most desirable. The writer revised his questionnaire and prepared the format as shown in Appendix D. In close cooperation with Dr. Cathcart of the Faculty of Education of the University of Alberta, the final format and wording of the questions were established and the questionnaire as shown in Appendix D was arrived at.

CHAPTER IV

EXPERIMENTAL DESIGN AND PROCEDURES

Introduction

In the previous chapter the design and the development of the instruments were discussed. The writer will discuss in this chapter the analysis of the data obtained with the instruments just mentioned. The calculation procedures will be discussed also. As the writer was mainly interested in describing what happened these procedures were different from the procedures used by others with a similar instrument. To be able to describe the phenomenon observed the writer found it helpful to represent the findings graphically. In order to be able to develop useful graphs the writer developed calculation procedures for that purpose.

In this chapter the writer will also describe the experimental design and procedures. A companion thesis by Westrom has been based on the development of the experimental design, therefore the description of the design will be kept to a minimum. The reader is urged to consult Westrom's thesis for greater depth and detail of the design.

First the design of the treatment and procedures will be discussed. Then follows a discussion of the analysis of the data obtained through the use of the Observation Schedule, the Log Sheet and the Questionnaire. The chapter will be concluded with a section on the calculation procedures.

<u>Design of the Treatment</u>

In order to be able to make any reasonable comparisons between the experimental group and the control group it was necessary that both groups should have the same availability of prepared reference material. Thus materials for the experimental group were prepared which were intended to serve the same function as the textbook did for the control group. A sample of the prepared materials appears in Appendix A.

The underlying structure of the material will be discussed next. The materials were developed on the basis of the broad objectives as developed by the Department of Education, Province of Alberta. Taking these objectives and goals the course was broken down into units called "topics" in this study. Each topic was further broken down into three parts, namely a "phase I", a "phase II", and a set of "Challengers". Phase I took the pupil through the entire development of a topic at an intermediate level while phase II was intended as a remedial program for the basic level and intermediate level pupils and an enrichment program for the advanced level pupils. The set of challengers were a set of non-routine problems. They were designed to give the pupil a chance to apply what he had learned up to this point. The problems required the higher mental processes as stated in Bloom's Taxonomy. (See Figure IX, page 51.)

To make phase I manageable it was once again divided into sections and in turn each section was broken down into one or more related objectives. Each objective was stated in behavioral terms and with each objective a sample of a possible examination question

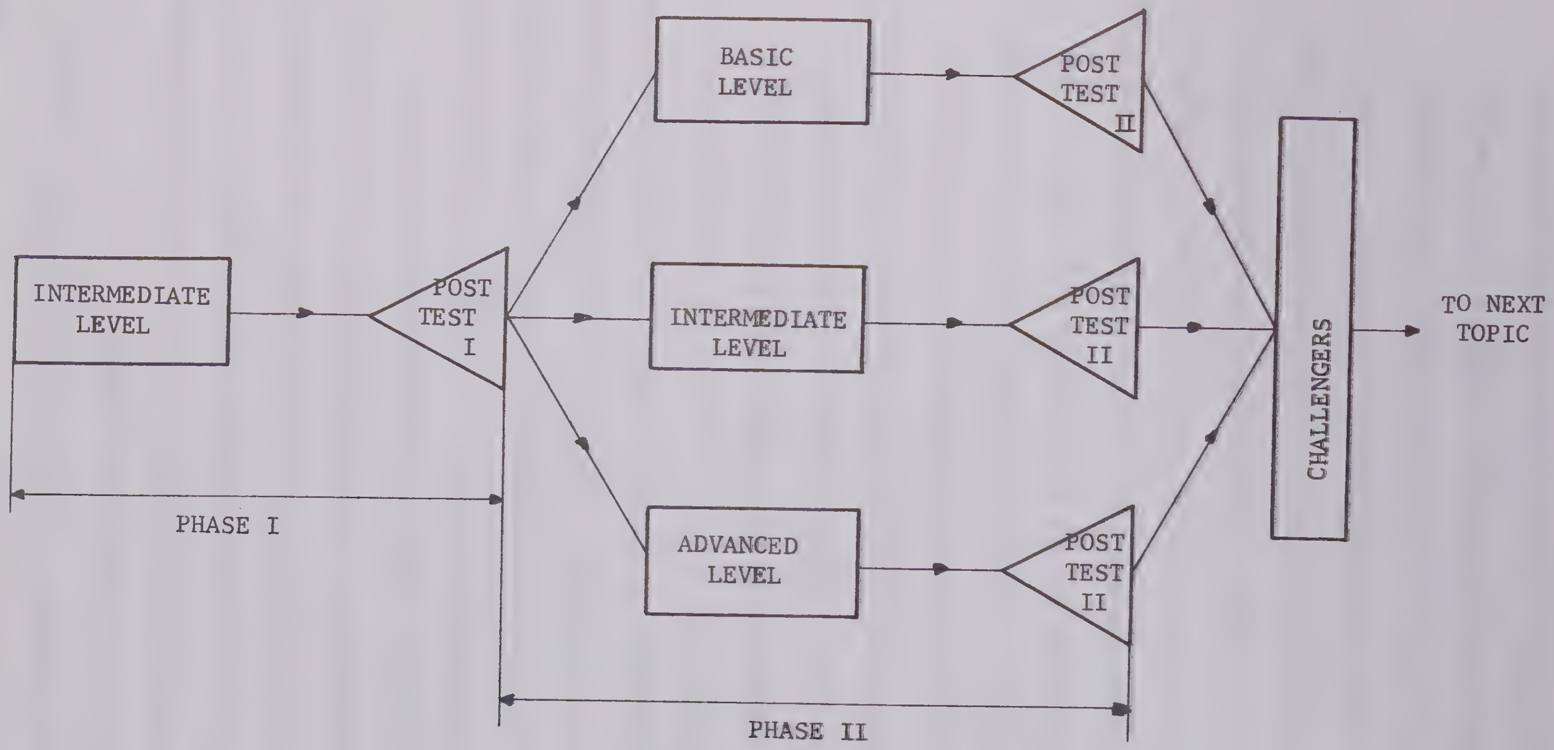


FIGURE IX

SCHEMA FOR A TOPIC AS COVERED

BY THE EXPERIMENTAL GROUP

and its solution were given. Also the level of competency required to achieve the particular objective was stated.

It was possible for some objectives to be written at different levels of difficulty, for instance the same objective could be used for basic level material, intermediate level material, and advanced level material. Wherever this was possible the basic level objective and the intermediate level objective were both stated in phase I. The intermediate level objective appears in italics. For each objective a description had been given which appeared on the white pages immediately following the pink page(s). Development related to an intermediate level objective also appear in italics. Then following the development was a set of applications for practise related to the specific objective(s). This set appears on the buff pages immediately following the white pages. The first question of each set of applications was a test-item on which the pupils checked themselves. The purpose of this check exercise was to tell the student that if he could do it there was no need for him to spend any more time on the materials in that section, but that he should go on to the next section. The answers to the check exercise and the remaining applications in this section appeared on the green pages at the end of phase I. If the pupil did not achieve the check exercise, then he was referred to a set of exercises which would help him to study the objective further. Following this set a check exercise appeared once again. If the pupil was successful on this exercise then he was directed to the next section. On the other hand if he was not successful then the pupil was directed to three further sources of help, namely a reference text, a student-helper, or his teacher. It was left up to the student

to consult whichever source he preferred. The foregoing procedure was repeated for each section. Thus it was possible for each pupil to proceed at his own rate. At the end of phase I the pupil was asked to study a vocabulary section as well as to do a set of review exercises before he proceeded to post-test I. On the basis of post-test I the pupils proceeded in phase II at the basic level, the intermediate level or the advanced level. The pupils in the basic level only studied basic objectives, those designated by a B notation. These objectives are the minimum level at which a pupil can successfully complete the course. The pupil was given a new set of exercises relating to the basic objectives only and he was directed through the topic once again in the same manner as in phase I. The pupils at the intermediate level received a packet containing instructions and were directed to review the same material they had studied previously in phase I. The pupils in the advanced level received for phase II a packet containing only advanced level objectives relating to material in phase I. All pupils were required to attempt only those objectives in phase II which they did not achieve in phase I.

At the end of phase II the pupils wrote post-test II. A different test was set for each level. After post-test II pupils were required to work for a few days on the challengers before they could go on to the next topic. To help the pupil to find his way through the material a flow chart was developed. The flow chart also showed how each pupil progressed through the material and gave the teacher an immediate picture of what the pupil was doing. Then for each pupil a record page was provided which showed him which objectives he was to study in each phase as well as at which level he was achieving

and what challengers he attempted. It also showed which challengers he had completed. In addition to the choice provided in the materials, the pupil also had a choice of independent study or a teacher-taught class.

The foregoing was a brief description of the design on which the study was based. For more detail the writer once again refers the reader to the companion thesis by Westrom.

Procedures

The foregoing brief description of the materials and their use was necessary for the writer has to describe the teacher's role in a setting where these materials were used. The writer observed the teachers in action in the independent study setting and the teacher-taught setting. He also observed the teachers in the control group in their classroom setting. In the following paragraphs and chapters the terms "independent study setting" and "teacher-taught" refer to two distinct phases of the experiment: the former referring to the situation where pupils work on their own and the latter to the class conducted by the teacher. The statement "regular teacher taught" refers to the classroom setting as found in the control group.

Three kinds of data were collected for each teacher in both the experimental group and the control group. First, each teacher was observed a number of times and his classroom actions were recorded using the Observation Schedule. The experimental group were observed three times each in the independent study classes and twice each in the teacher-taught classes. The teachers in the control group were observed four times each except one who was seen three times only

because of illness. Secondly, each teacher was asked to keep a log sheet for the duration of a week at a time. Three randomly selected weekly samples were used. Because of illness one teacher in the control group was unable to complete them and the others in the control group lost part of a week because of examinations and other activities beyond the control of the writer and the teachers. So only two weekly log sheets were available for the remaining two teachers of the control group. Thirdly, a questionnaire was completed by each teacher in both the experimental and the control group. Again one of the teachers in the control group was unable to complete the questionnaire, because of his ill health.

Analysis of the Data

The data gathered on the Observation Schedule were used for both the experimental group and the control group to describe the following areas: checking of assignments; formal teaching; directed practise; managerial tasks; testing; seat work; grouping of pupils; classroom climate, which is the atmosphere of the study group observed; and the affective teacher talk and teacher actions. The data were analyzed in terms of frequencies per group, which were expressed as a percentage so that the experimental and control group could be compared in relation to a common measure. The frequencies referred to above are actually the frequencies of observation periods during which the particular behavior had been observed. For detailed analysis of the data see the next section in this chapter called "Samples of Calculation Procedures".

The data obtained from the log sheet were processed on the basis of the experimental group and the control group. The data were analyzed with respect to time spent on different classroom activities and with respect to time spent on different activities related to the teaching of the grade seven mathematics. For detailed analysis see the next section.

The questionnaire was analyzed on the basis of how frequently a certain group gave a particular response. The responses were used to give a descriptive analysis of the teachers' reactions.

Samples of Calculation Procedures

The writer faced a problem once he had obtained the data with the Observation Schedule. He had different numbers of teachers in the experimental group and the control group. To compound the problem he had for each of the three groups -- the independent study, the teacher-taught, and the regular teacher-taught group -- a different number of observations because of physical limitations and lack of time. In order to be able to make comparisons the writer used the following scheme, which he will illustrate with a sample section of the schedule. The section on seat work will be used. For the sake of brevity only the numbers referring to the respective behaviors on the schedule will be used. For teacher 2 the three observations were for this section as follows:

BEHAVIOR	RESULTS OF OBSERVATION I				RESULTS OF OBSERVATION II				RESULTS OF OBSERVATION III				GRAND TOTAL
	I	III	V	Tot	I	III	V	Tot	I	III	V	Tot	
6.1	-	-	-	0	-	-	✓	1	-	-	✓	1	2
6.2	-	-	-	0	-	-	-	0	-	-	-	0	0
6.3	-	-	✓	1	-	-	-	0	-	-	✓	1	2
6.4	-	-	-	0	-	-	-	0	-	-	✓	1	1
6.5	-	-	-	0	-	-	✓	1	-	-	✓	1	2
6.6	-	-	-	0	-	-	-	0	-	-	-	0	0
6.7	-	-	-	0	-	-	✓	1	-	-	-	0	1
6.8	-	-	-	0	-	-	-	0	-	-	-	0	0
6.9	-	-	-	0	-	-	✓	1	-	✓	✓	2	3
6.10	-	-	-	0	-	-	-	0	-	-	-	0	0
6.11	-	-	-	0	-	-	-	0	-	-	-	0	0
6.12	-	-	-	0	-	-	✓	1	-	-	✓	1	2
6.13	-	-	-	0	-	-	-	0	-	-	-	0	0
6.14	-	-	-	0	-	-	-	0	-	-	-	0	0

FIGURE X

SAMPLE OF TABULATION FOR OBSERVATION SCHEDULE

For each of the observations the writer has shown how the entries appeared on the schedule. The entries for each visit have been totalled and these totals have been added to give the grand total. Thus a grand total for each behavior in section six has been obtained. In the same manner the grand totals for the other teachers for this section have been obtained. The grand totals for all teachers and all sections have been reported in Appendix E.

Next the writer tabulated all the grand totals for the teachers and used these to form group totals. In the next table it has been shown how this was done. The values shown in the total column are the group totals referred to previously. These totals are actually the total number of five-minute observation periods during which the

SEAT WORK

TEACHER BEHAVIOR	CONTROL					INDEPENDENT STUDY							TEACHER-TAUGHT CLASS						
	1	2	3	Tot	%	4	5	6	7	8	Tot	%	4	5	6	7	8	Tot	%
6.1	1	2	3	6	18	3	2	6	8	8	27	60	-	-	-	-	-	0	0
6.2	-	-	-	0	0	1	-	1	-	1	3	7	-	-	-	-	-	0	0
6.3	1	2	-	3	9	-	-	4	1	4	9	20	-	-	-	-	-	0	0
6.4	-	1	-	1	3	6	3	7	8	8	32	71	1	-	-	-	-	1	4
6.5	1	2	2	5	15	6	4	8	8	8	34	77	1	-	-	-	-	1	4
6.6	-	-	-	0	0	-	1	-	-	-	1	2	-	-	-	-	-	0	0
6.7	1	1	-	2	6	4	1	2	2	4	13	29	-	-	-	-	-	0	0
6.8	-	-	-	0	0	-	3	-	3	-	6	13	-	-	-	-	-	0	0
6.9	1	3	2	6	18	-	-	-	-	-	0	0	-	-	-	-	-	0	0
6.10	-	-	-	0	0	6	1	7	8	8	30	67	1	-	-	-	-	1	4
6.11	-	-	-	0	0	-	-	-	-	-	0	0	-	-	-	-	-	0	0
6.12	-	2	-	2	6	5	1	2	2	9	19	42	-	-	-	-	-	0	0
6.13	-	-	1	1	3	-	-	-	-	-	0	0	-	-	-	-	-	0	0
6.14	-	-	2	2	6	-	-	-	-	-	0	0	-	-	-	-	-	0	0
					/33							/45							/27

FIGURE XI

SAMPLE OF CALCULATION OF PERCENTS

FOR EACH BEHAVIOR FOR EACH GROUP

indicated behaviors had been observed. Since the number of teachers within each group was different and since the number of observations were different for each group, it was convenient to express the total obtained as a percent of the total possible five-minute periods for the group during which the behaviors could have been observed. These totals have been shown at the bottom of each total column. The percents to the nearest percent have been shown in the percent column. By expressing the totals as percents it is possible to make a reasonable comparison between the groups. It is these percentages that will be used to make the graphs which form the basis for the descriptions in Chapter V. The example will be graphed as follows:

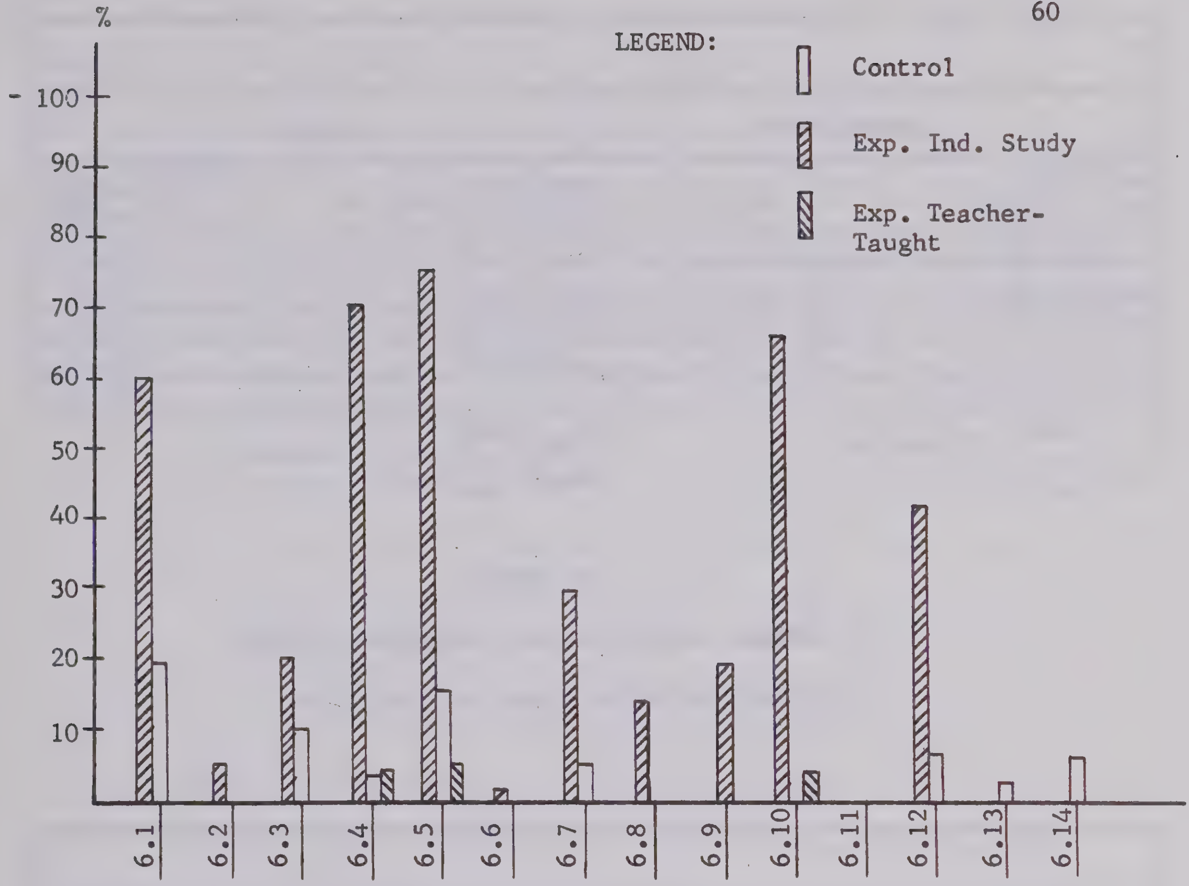
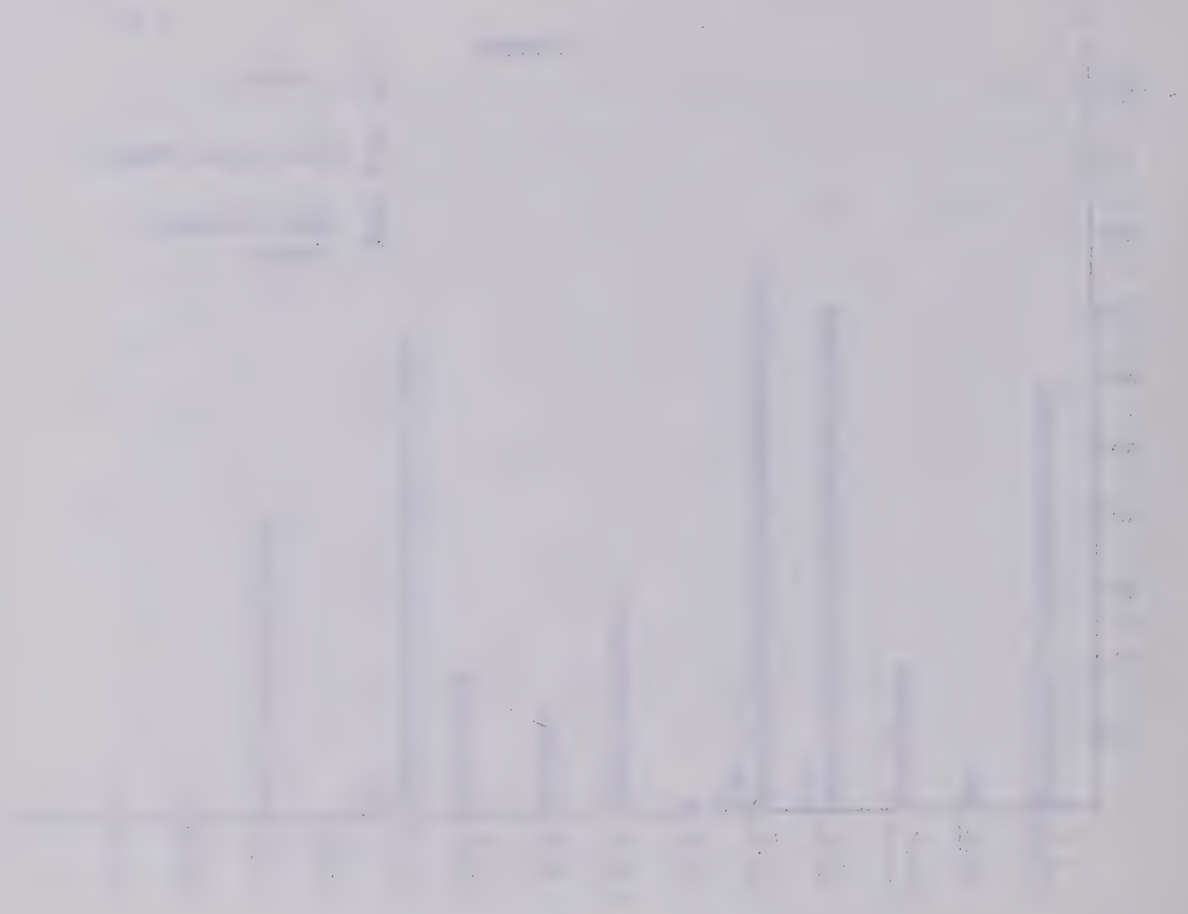


FIGURE XII

SAMPLE GRAPH BASED ON DATA OF FIGURE X

The above described procedure was used for the first nine sections of the observation schedule. For the tenth section a slightly different approach had to be used, because the group totals obtained are actual totals and are not dependent upon the actual number of five-minute observation periods. Instead of percent the behaviors have been expressed as statements per 100 minutes. The 100 minute interval was chosen so that graphing would be easier.

Next, the writer will describe the method of calculation used for the analysis of the data obtained from the log sheets. For the



	CONTROL					EXPERIMENTAL							
TEACHER / TIME INTERVAL	2	4	Tot	AV/ W/T	Time /W/T	1	5	6	7	8	Tot	AV/ W/T	Time /W/T
1-15	2	1	3	0.75	5.6	-	-	-	-	4	4	0.27	2.0
15-30	-	1	1	0.25	5.6	-	9	1	1	-	11	0.73	16.5
30-45	1	2	3	0.75	28	-	1	4	1	-	6	0.4	15.0
45-60	-	1	1	0.25	13.1	-	-	-	-	-	-	-	-
over 60	-	-	-	-	-	2	-	-	2	2	6	0.4	36.0
		Divisor	4	Tot. in Min.	52					Divisor	15	Tot. in Min.	70

FIGURE XIII

SAMPLE TABULATION FOR ACTIVITIES RELATED TO

THE TEACHING OF GRADE SEVEN MATHEMATICS

	CONTROL					EXPERIMENTAL							
TEACHER / TIME INTERVAL	2	4	Tot.	Av/ Class	Time/ Cl.	1	5	6	7	8	Tot.	Av/ Class	Time/ Cl.
1 - 10	2	2	4	0.15	0.8	-	1	-	-	-	1	0.01	0.1
10 - 20	2	1	3	0.11	1.7	1	-	-	-	-	1	0.01	0.2
20 - 30	3	4	7	0.26	6.5	2	1	1	3	2	9	0.08	2.0
30 - 40	1	1	2	0.07	2.5	10	20	10	6	8	54	0.5	17.5
40 - 50	-	-	-	-	-	3	4	2	5	3	17	0.16	7.2
		Divisor	27	Tot. in Min.	12					Divisor	108	Tot. in Min.	27
				24%								42%	

FIGURE XIV

SAMPLE TABULATION FOR CLASSROOM ACTIVITIES RELATED TO

THE TEACHING OF GRADE SEVEN MATHEMATICS

analysis tables as shown in the samples on page 61 were used. These tables were used to tabulate the frequency with which each teacher had marked each time interval. Tables similar to the one shown in Figure XIII were used for each of the eight categories dealing with activities related to the teaching task. Tables similar to the table in Figure XIV were used for each of the eight classroom activities. First the writer will discuss the use of the top table. When the teachers marked the log sheets they entered a number of checks for different time intervals for each category during a single week. Thus per teacher per week a frequency for each time interval appears. When these frequencies for the three different weeks are combined a total frequency for each time interval in each category is obtained. These total frequencies for each teacher have been reported in the top table.

The writer again faced the same problem as he experienced with the observation schedule, because again he had an unequal number of teachers in each group and he had an unequal number of log sheets for each group. To be able to come to some manner of comparison the writer tabulated the average frequency per teacher per week for each time interval. As stated previously the frequencies entered for each teacher were total frequencies for that teacher. Now the total frequencies for the teachers within a group were added to give the total frequency for the group for each time interval. These total frequencies for the group appear in the total column in the table. To obtain the average frequency per teacher per week the total frequencies for the group were divided by a divisor which was obtained by multiplying the number of teachers within each group by the number of log sheets per teacher

for that group. Now the average time intervals per week per teacher were known, but the question remained, "What is exactly the average time interval?" The answer is that on the average per week each teacher within a group spent a given portion of a particular time interval on a particular task. For example if the average is 0.75 for the 1 - 15 minute interval, then this means that each teacher within that group on the average spent 0.75 of that 1 - 15 minute interval on a given task per week. 0.75 of a time interval does not mean much unless it can be expressed in actual minutes, for then the results for the various intervals can be added so that a total per week per teacher can be obtained. To convert 0.75 of the 1 - 15 minute interval the writer assumed the mean of the interval which is 7.5 minutes and so 0.75 of the interval became 0.75 of 7.5 minutes which is 5.6 minutes. Similarly 0.75 of the 30 - 45 minute interval is 0.75 of 37.5 minutes which is about 28 minutes. Thus the average time intervals were converted into minutes. These minute values were totalled for each group and a total time per week per teacher was obtained for each of the eight categories about activities related to the teaching task.

A slight difficulty presented itself with the open-ended interval of over 60 minutes. It was difficult to ascertain what average minute value should be assigned to this interval. Hence the writer went back to the school and discussed this aspect with the teachers. It appeared that if the writer took 90 minutes as the mean time for this interval that he was close to the actual average time spent by the teachers. It is difficult to attain exact figures unless someone keeps exact time. It is almost impossible for the classroom teacher

to do so unless he neglects the job he is supposed to do, that is teaching.

For the bottom table the figures in the total column were obtained in the same manner as described for the top table. The divisor was obtained in a different manner and as a result the averages are different. The divisor is the total number of class periods that each group reported on. Thus the average obtained for each time interval now is an average per class period. These average time intervals per class period were translated into minute values in the same manner as was done for the previous table. Thus total times in minutes for each activity per class were obtained.

The two groups taught classes of different lengths hence adjustments had to be made in order to be able to compare the two groups. Thus each set of figures was expressed as a percent of classtime. The results thus obtained for each of the eight categories for each group was graphed. The graph appears in the discussion in Chapter V. The actual frequencies obtained for this study appear in the tables in Appendix F.

The description and analysis of the results of the questionnaire was based on the yes - no response frequencies. These frequencies will be shown in the tabulation table in Appendix G.

The above discussed calculation procedures were employed to obtain the results discussed in the next chapter. Chapter V is the report of the actual findings and their interpretations.

CHAPTER V

RESULTS, INTERPRETATIONS AND CONCLUSIONS

Introduction

This chapter contains the final results as shown in the series of graphs based on the findings of the three different instruments used. The findings based on each instrument will be discussed separately. In the summary section of the chapter, interpretations will be given and conclusions will be drawn based on the data obtained from the three instruments.

Findings From the Observation Schedule

The results of the observation schedule were analyzed under ten different headings. The areas that were dealt with were: checking assignments, formal teaching, directed practise, managerial tasks, testing, seat work, laboratory work, grouping, classroom climate, and affective teacher talk and actions. The individualized group was considered in two different settings, namely in the individualized setting and in the teacher-taught setting. At the same time the teachers in the control group were considered as well so that a comparison with a regular teacher-taught situation would be possible. Graphs have been used to report the results and to assist the reader in gaining a better overall idea of the teacher's role in the experimental program. The data on which the graphs have been based can be found in Appendix E.

It should be noted again at this point that the percentage

figure shown on the graph does not refer to percent of class time, but to the percent of five minute observation periods during which a particular behavior shown by one or more of the teachers was observed in a given setting, e.g. the experimental group was observed for a total of 45 five minute periods with respect to any of the behaviors shown in the first nine parts of the schedule, i.e. 45% means that in 45% of the 45 five minute observation periods a particular behavior had been displayed once or more than once by one or more teachers in the group. The criterion used has been to report what occurred for a group as a whole not what happened in a given class at a given time. The number following the headings of each of the subsections corresponds to the number on the Observation Schedule.

Checking Assignments (1.0)

A glance at the graph on page 67 shows the experimental teachers did practically no checking of assignments. Only a very minimal comment with respect to the doing of homework was observed in the individualized setting. The comment was made by only one of the teachers involved in the project. It could be concluded then that no formal checking of assignments took place in the experimental program in contrast to what was being done by the control group. The control group did check assignments regularly and used a number of variations to do the checking.

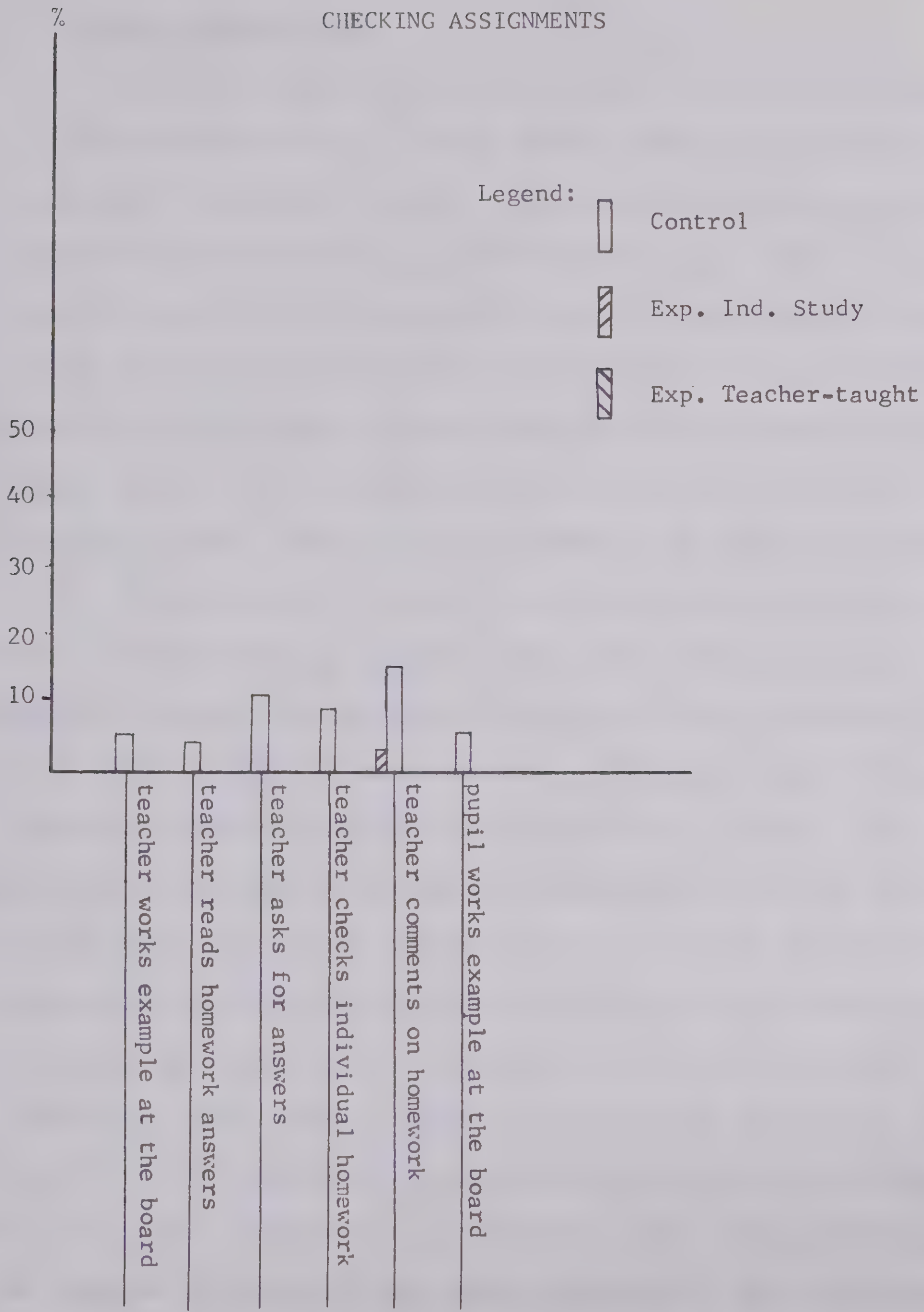


FIGURE XV

GRAPH SHOWING PERCENTAGE DISTRIBUTIONS

FOR BEHAVIORS RELATED TO CHECKING ASSIGNMENTS

Formal Teaching (2.0)

Under this heading the writer looked for what would normally be considered the routine a teacher follows when he explains and teaches new material and concepts. In other words the teacher is the more dominant figure at this particular time of the lesson. From the graph on page 69 it can be seen that basically this approach to teaching was not used in the individualized classroom setting. The few small entries that have been shown refer to the few minutes each teacher usually took at the beginning of the period to explain what was to be done next and what changes were to be made in the routine and materials, if any. It was generally intended to clear up any confusion that might exist or difficulties of a general nature that might have arisen. Basically this was the few minutes used to do a quick housekeeping job in order to keep the pupils working efficiently. There is quite a contrast between the teacher-taught class as it was used in the experimental setting and the regular teacher-taught class as used by the control group. In the teacher-taught class of the experimental approach the teacher used the board while lecturing and questioning for almost the entire period. The teacher in the control group lectured and questioned for slightly less than half the period. The behavior of the experimental group was basically due to the program, for the teacher-taught class was intended to be a lesson reviewing the material which a pupil had studied previously. The pupil attended this class to hear a teacher's exposition of the material he had tried to read and understand earlier. The emphasis in this class, as a consequence, was on explaining (lecturing), showing (using the board),

FORMAL TEACHING

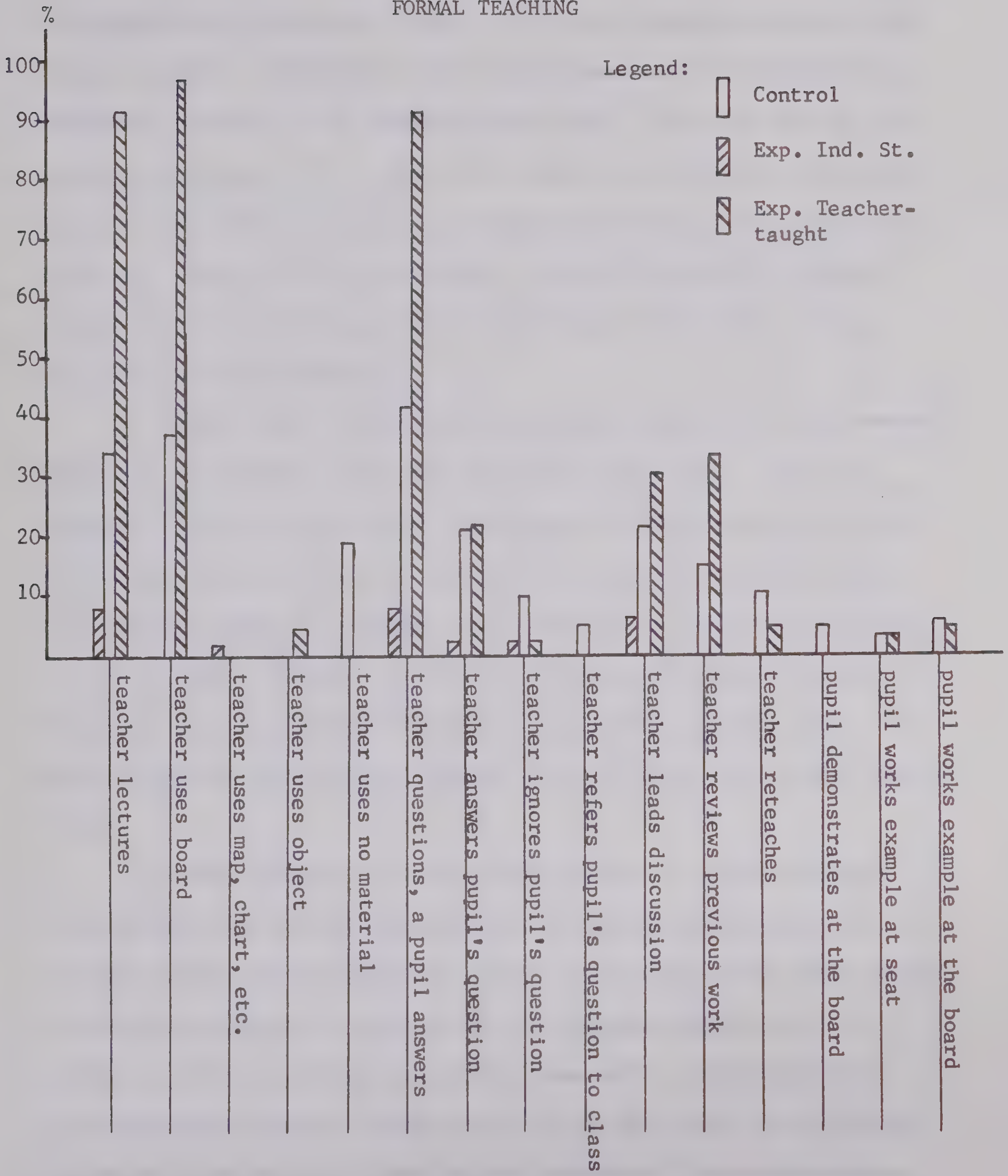


FIGURE XVI

GRAPH SHOWING PERCENTAGE DISTRIBUTIONS FOR

BEHAVIORS RELATED TO FORMAL TEACHING

and questioning the pupils to see if they were grasping the ideas that were discussed. In contrast the control group used this approach to teach the material to be learned by the pupil. The pupil had not had a previous attempt at it. Thus the teacher in the control setting had to give more time to seatwork and related activities. The graph bears this out. Therefore the difference in purpose between the teacher-taught classes in the experimental setting and the control setting did show up very dramatically.

It was rather interesting to note that neither the experimental group nor the control group used any audio-visual aids. This was expected to some extent in the experimental setting because the film-strips were made available to the individual pupils or small groups of pupils and were not intended to be used by the teacher to instruct an entire class. Because of the individualized setting in the experiment, the use of an overhead projector was limited at this time. The most surprising part was the absence of any of these aids in the control group.

A final comment to be made with respect to formal teaching. Despite the fact that in the experimental teacher-taught group the teacher talked almost the entire period and in view of the class loads, which ran from about 15 to about 55, the teachers experienced no difficulty in maintaining discipline. When 40 or more pupils were in attendance a number of them had to sit on the window sills and on the floor which created a very crowded condition. The pupils mainly attended to get help and information. Hence the pupils were in the teacher-taught class for a purpose, which perhaps partly explains the absence of disciplinary problems. More on discipline will follow

in a later section.

Closely linked with formal teaching is the idea of directed practise, which will be discussed next.

Directed Practise (3.0)

The writer is interested in this section with the practise a number of teachers use as a follow-up to a formal presentation. The teacher gives one or two examples for the pupils to work on to ascertain if the pupils have grasped the ideas that have been presented in the lecture. The writer found the experimental group did not employ this concept in either the independent study setting or in the teacher-taught setting, however the control group did use it. The graph on page 72 refers to this aspect and shows that only the control group made use of directed practise. Following directed practise the regular classroom teacher usually gave an assignment and time for the class to work on the assignment. The writer has labeled this aspect as seat work. This will be discussed in the next section.

Seat Work (6.0)

Seat work in the regular classroom is usually given to provide an opportunity for the pupil to work on his own and do the applications in relation to the theory and development given previously in the lesson. This procedure gives the teacher the opportunity to help individual pupils or small groups who still experience difficulties. Seat work, of course, is a major part of any form of individualized learning. It should thus appear quite prominently in the independent study setting. The graph on page 73 relates to seat work. It is rather interesting to note that the experimental group did not use it in

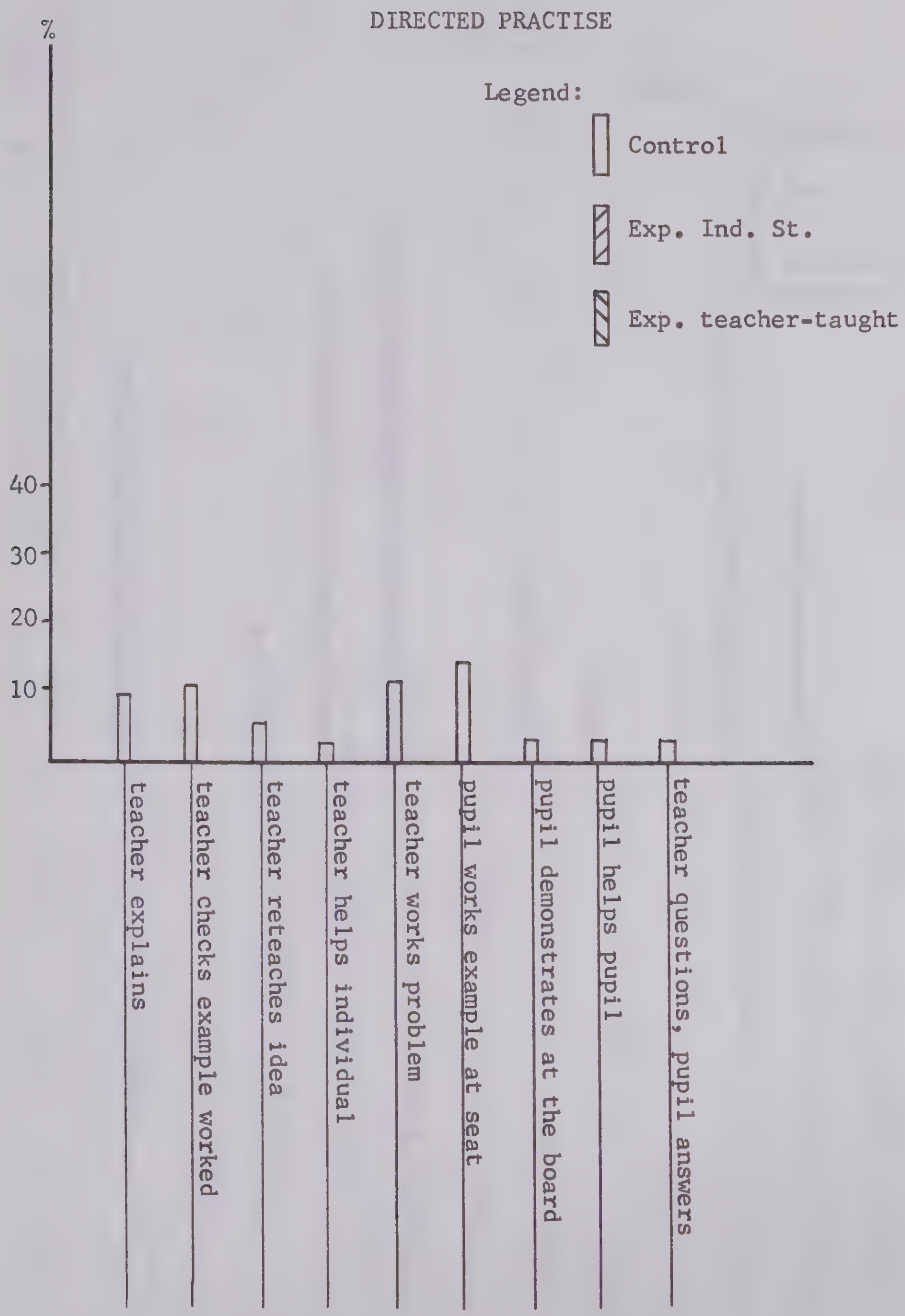


FIGURE XVII

GRAPH SHOWING PERCENTAGE DISTRIBUTIONS

FOR BEHAVIORS RELATED TO DIRECTED PRACTISE

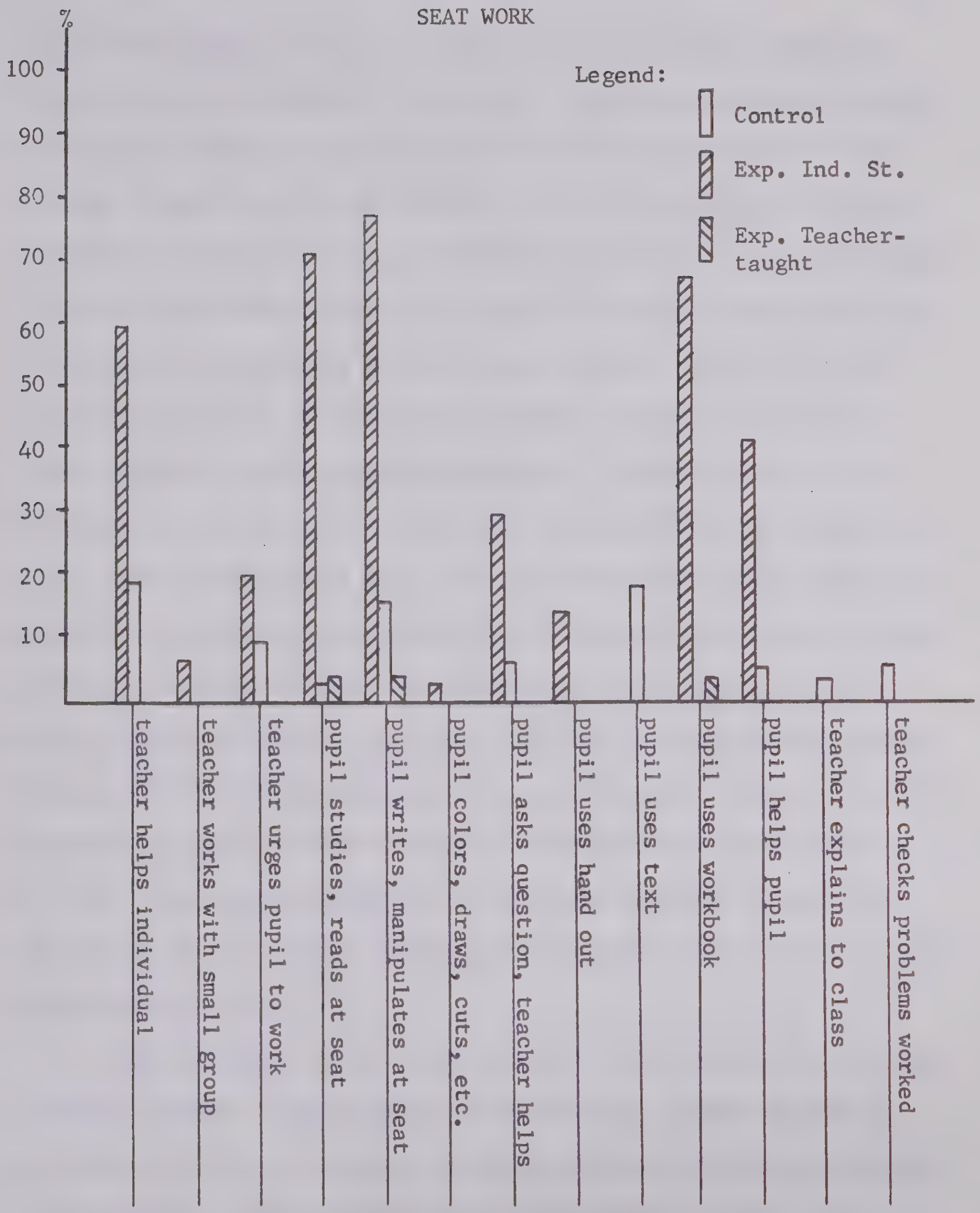


FIGURE XVIII

GRAPH SHOWING PERCENTAGE DISTRIBUTIONS FOR

BEHAVIORS RELATED TO SEAT WORK

the teacher-taught setting. As was to be expected the independent study setting used seatwork extensively. This was necessary in order to free the teacher so he could help the individual pupil and also to make it possible for the individual pupil to move at his own rate through the materials. As can be seen from the graph the teacher spent a considerable amount of his time helping individual pupils while the other pupils were sitting in their desks working. The pupils were never observed using an authorized textbook. They seemed to rely almost entirely on the prepared materials. Another aspect of the independent study setting was that pupils were helping each other.

The control group used seat work relatively little. Yet proportionately much the same distributions appeared with respect to such activities as teachers helping individuals, pupils helping other pupils, pupils working on their own, and pupils using the text books. In both settings the teachers had to urge the pupils from time to time to work. A couple of aspects which the experimental group did not do were: the teacher going through the room checking the work of the pupils and the teacher stopping the class and explaining difficulties to the entire class.

In conclusion it can be stated that the findings show that the intended purpose of the program can be realized through the use of seat work and that the regular classroom teacher could spend more time with individual pupils if more time was available for seat work. This of course shows the need for a form of individualized instruction. Up to this point the behavior of the teacher in the independent study setting has been found considerably different from the behavior of the

teacher in the control setting. The writer will summarize these differences later in this chapter.

In relation to a particular classroom procedure certain groupings of pupils are used. The next section has been devoted to the grouping of pupils within a classroom as observed in both the experimental and the control setting.

Grouping (8.0)

Teachers can group the pupils in a given class in many different ways. They might group according to ability, interest, age, sex, attitude, or materials to be used. The grouping might be done by the teacher or the grouping can be left to the pupil himself. The writer refers to grouping by the teacher when he uses the statement "administrative grouping". On the other hand the writer uses the concept "social grouping" to mean a grouping arranged by the pupils themselves. First the writer will discuss the findings with respect to "administrative grouping".

From the graph on page 76 it appears that the teacher in the independent study setting spent considerably less time with the entire class, administratively grouped, than did either the teacher in the experimental teacher-taught class or the teacher in the control class. The main reason for handling the class as a single group was to be able to explain to the class some changes in procedure or to review some aspects of the method. As was pointed out in a previous section this form of grouping was used only to do some housekeeping to ensure the efficiency of the program. From the graph it also becomes clear that other forms of formal grouping (administrative grouping) in the

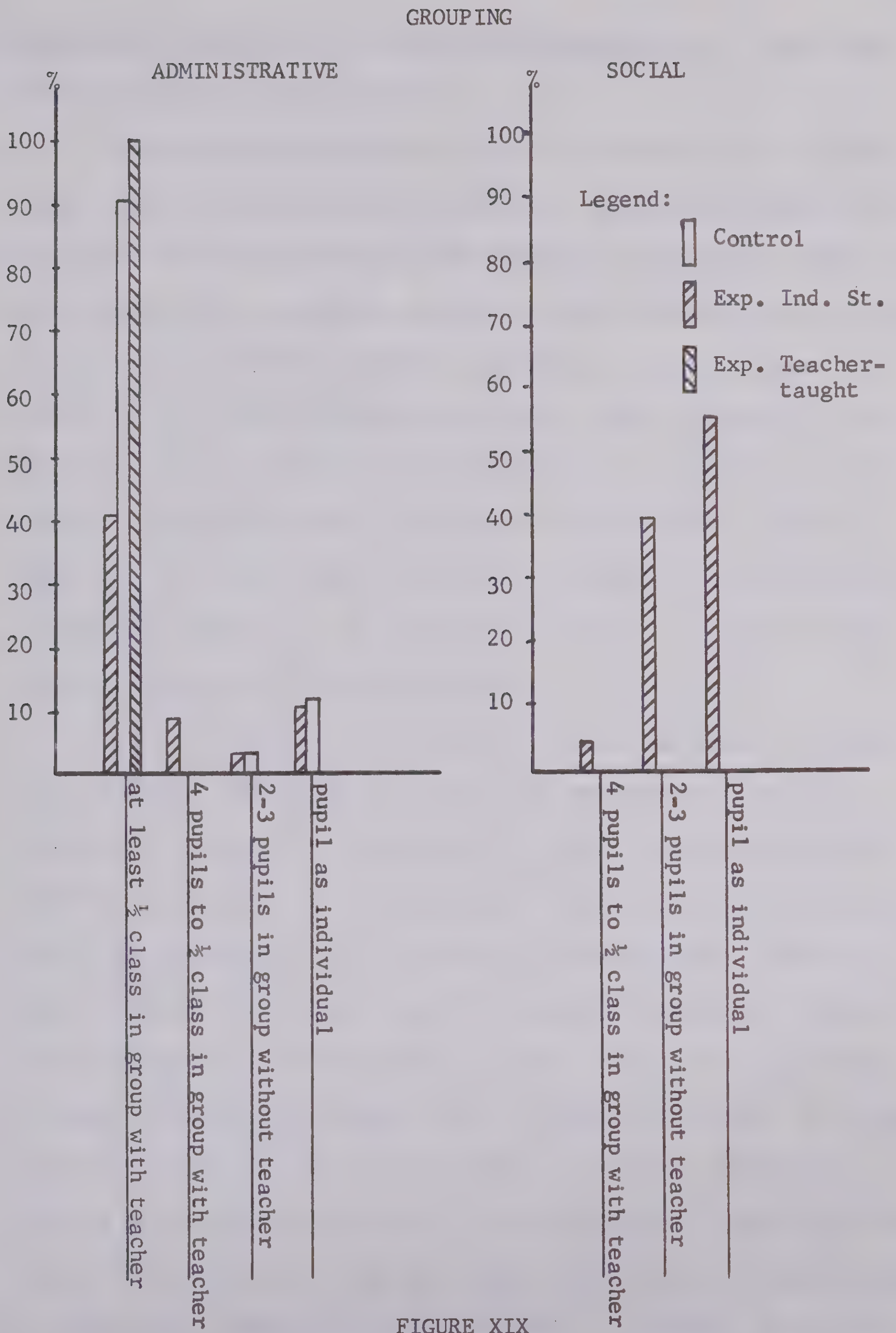


FIGURE XIX

GRAPHS SHOWING THE PERCENTAGE DISTRIBUTIONS

FOR BEHAVIORS RELATED TO "ADMINISTRATIVE" AND "SOCIAL" GROUPING

independent study setting did not differ appreciably, if any, from those used by the control group.

The teachers used only one form of grouping in the teacher-taught class in the experimental setting. They had the entire class as a group with the teacher for the duration of the class period. This was pointed out in a previous section on formal teaching and, in that respect, the two findings support each other. In this respect the teacher in the teacher-taught class did not differ appreciably from the control group except that the teachers in the control group did provide for some seat work during which time the pupils worked on their own. Still the control group kept the pupils "administratively" grouped as individuals. No freedom of grouping on the part of the pupil was observed in the control group.

Next the writer will discuss the "social grouping" of pupils in a classroom. The graph shows that the independent study group permitted this type of grouping, but the other two settings did not allow this grouping. It is of interest to note that pupils preferred to work as individuals, for this could be observed most frequently. Groups of two or three pupils working together without the teacher could be observed quite frequently as well. The last two groupings mentioned coincide very closely with the observations made in the seat work section with respect to the number of times the teacher was observed helping individual pupils and the number of times pupils were observed helping pupils. The only time larger groupings were observed the pupils were found to be with a teacher. It appears that pupils with a common problem would approach a teacher together or when the

pupils found a teacher dealing with a problem they had, the pupils voluntarily joined the group. The latter type of grouping was not observed to any great extent.

Closely linked with the foregoing behaviors discussed up to this point is classroom climate. This will be discussed in the section following.

Classroom Climate (9.0)

Classroom climate is the general tone of the classroom. The writer hoped to be able to describe what the general feeling in the classroom was by looking at a number of specific behaviors displayed by the teacher and by the pupils. Often the behaviors of the pupils show indirectly the task a teacher faces and how the teacher appears to be handling the situation. The diagram following shows these behaviors and the frequency of their occurrence with respect to observation periods. A number of behaviors displayed by the independent study group differ significantly from those displayed by the other two groups. In the independent study setting the writer noticed that the teacher moved about less freely than he did in the other settings. On the other hand the pupils had more freedom to move about in the independent study setting. The latter two behaviors complement each other. In the independent study session the pupils come to the teacher for help or go to other pupils for help. As soon as pupils come to the teacher, the teacher loses freedom of movement and hence was seen more frequently in the same location in the classroom. All the pupil movement was not due to going for help to a teacher or another pupil. A number of pupils had to leave the room to go to another

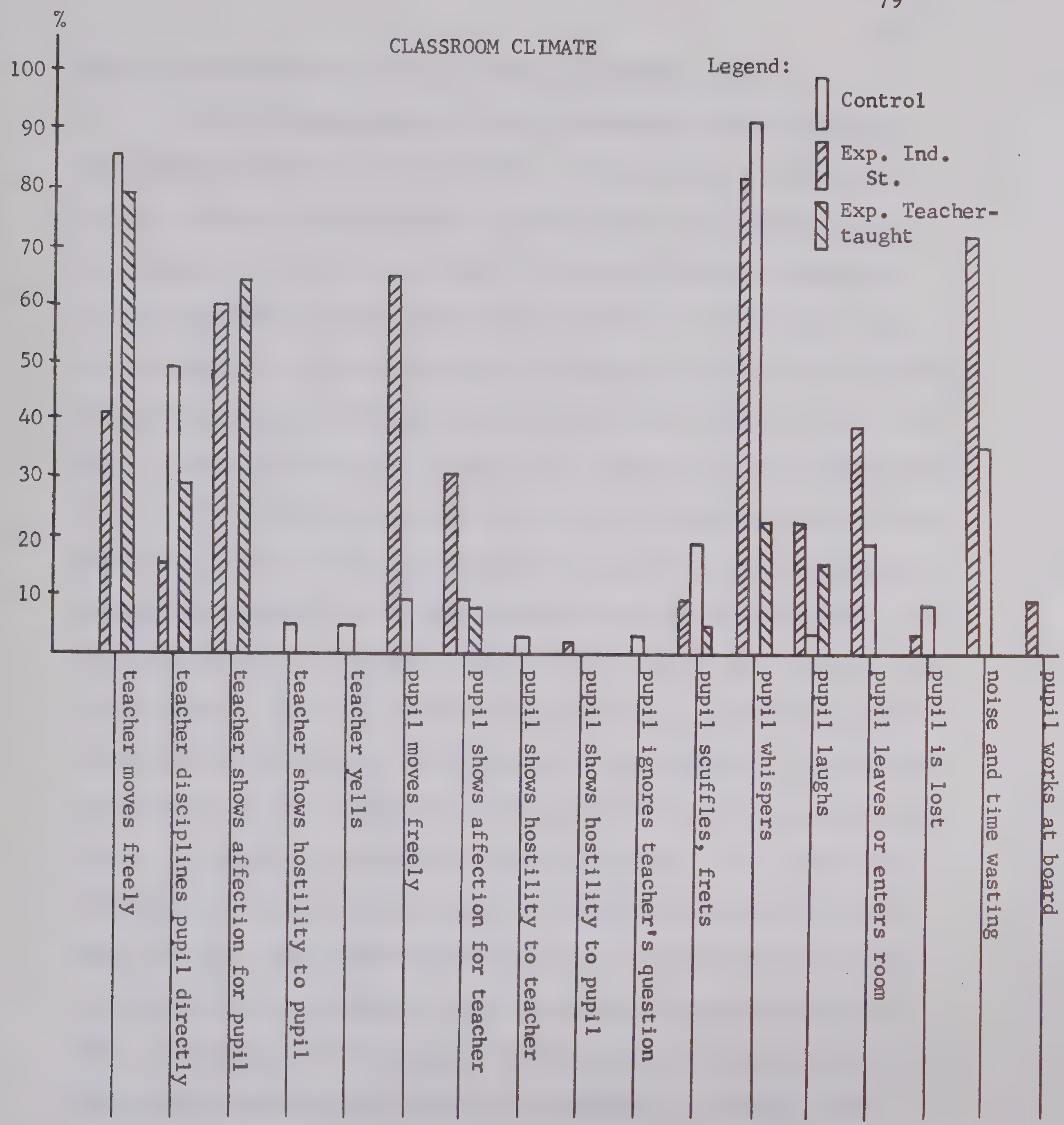


FIGURE XX

GRAPH SHOWING PERCENTAGE DISTRIBUTIONS FOR

BEHAVIORS RELATED TO CLASSROOM CLIMATE

room to either write a test or consult a reference book.

Since working together in the independent study setting was encouraged the teacher had to tolerate a fair amount of whispering. It was somewhat surprising that the same behavior was observed just as frequently in the control group. The only difference between the two settings was that more pupils were involved in this behavior in the independent study setting than was the case in the control setting. The teacher in the independent study setting had to make sure that the level of noise did not get too high. The writer had noted several times in the remark section on his schedule that the noise level was low and that mainly time wasting and socializing took place. The last behavior became very apparent in the individualized setting and the teacher had to adjust to it. It is quite possible pupils wasted just as much time in the control class, but in the latter setting it could not be observed as readily since the pupil could pretend to be attentive. At any rate the teachers of the independent study group seemed to have good discipline. The number of times they had to discipline a pupil were considerably less than in any of the other settings. Of course in the other settings even whispering could not be tolerated when the teacher was lecturing, and consequently the pupils were disciplined for this. More interruptions for all pupils occurred, because the teacher in formal teaching situations had to stop the development of a concept every time he disciplined a pupil. This leads to loss of continuity and, the writer felt, a wasting of time. In the section on managerial tasks it will be noted that the teacher in the independent study group did not discipline the entire class as often either as did the teachers

in the control group. The discipline appears to be easier for the independent study group, but in the section reporting on the findings of the questionnaire it will be noted to be more difficult according to the feelings expressed there by the teachers. With less actual disciplining occurring, less hostility built up between the teacher and the pupils. The teacher seemed to be more inclined to show his affection for the pupils in the independent study group than the teachers in the control group. As the graph bears out, this behavior was observed about twice as frequently in the independent study setting as it was in the control setting. This also seemed to lead to the pupils showing their affection for the teachers. Again this behavior was observed much more frequently in the independent study setting than in the control setting. A note of caution must be struck here, for the observer had difficulty in recording the behavior just mentioned. The observer placed himself in a position at the back of the classroom so he could observe the teacher at all times. This position made it difficult to observe the facial expressions of the pupils in both settings. Yet it is probable that it was possible to observe affection more frequently in the independent study setting, because the pupil was given a better chance to express himself in a one-to-one relationship with the teacher. In the control setting the pupil was not given the same opportunity. Also coupled with less disciplining was the feeling of being more relaxed. The latter was borne out by the fact that laughter by pupils was observed much more frequently in the experimental group than it was in the control group.

Up to this point the writer has mainly shown the contrast

between the independent study group and the control group. The remaining behaviors shown on the graph for classroom climate, which were not discussed were not of great importance because of their infrequent occurrence. Perhaps the only significance these behaviors have is that they were observed in one setting and not in the other. However, due to the sampling method these behaviors could have been missed in one or the other of the settings. Thus these behaviors were not used to contrast the two groups.

Finally, the writer will mention just a few things about the experimental teacher-taught class. The teachers in this setting showed the same frequency of affection for pupils as they did in the independent study setting. Pupil laughter also occurred as frequently as it did in the independent study setting. On the other hand the teachers behaved in the experimental teacher-taught class the same as did the teachers in the control group with respect to moving about in the room. Whispering by pupils was observed considerably less frequently in this setting. This was perhaps due to the fast pace the teachers maintained in the presentation of the material. Also the teachers usually made it quite clear that talking on the part of the pupil could not be tolerated. They enforced this, especially when the class was large, and the pupils seemed to be quite eager to comply with this request.

In closing the writer should remark that the teachers in the experimental group were more inclined to show feelings towards their pupils and that as a consequence there seemed to be a more relaxed atmosphere in the classroom. The pupils seemed to be slightly more

relaxed as it manifested itself in more pupil laughter. The experimental group seemed to have created a better working atmosphere, which could be due also to differences in characteristics of the teachers in the experimental and control group.

Testing (5.0)

In this section the writer wishes to discuss briefly the differences in the testing programs as observed between the experimental and the control group. It should be noted at the outset that no testing was to take place in the experimental teacher-taught class and none was observed in that setting. Hence the testing in the experimental setting took place only in relation to the independent study setting.

In the experimental group testing occurred more frequently than it did in the control group. This was mainly due to the design of the experimental program. Quite frequently it was possible to observe pupils writing a test in the independent study setting. On certain days the pupils would leave the room to go to a central location to write the test. The supervision in that room was done by a teacher aide. Yet from time to time one or two pupils would be writing in the same room as the instruction was taking place. Whenever there were only a few pupils writing the teacher did not seem to supervise, he simply carried on helping individual pupils as before. If there were more than two or three the teacher seemed to supervise to prevent possible cooperation with other pupils. In general the teacher did not supervise as the graph on page 85 will show. Testing occurred quite regularly, but supervision by teachers was minimal. In the

control group testing did not occur on any of the days that the writer visited their classes and hence teachers in that group were not observed in the testing situation. The latter bears out the fact that the experimental group had a more extensive testing program than the control group.

In the experimental group tests were handed back and then discussed with each pupil individually. The writer could not establish a frequency with respect to this for it was impossible to tell from the position at the back of the room if the teacher was discussing a test or regular class work. In the control group however it was quite possible to observe the teacher discussing the test. The graph on the page following shows this clearly. The teacher discusses the test with the entire class at once. The writer was able to observe the marking of a test by the class in the control setting. In the control setting much of the diagnostic value of the test was lost, since the teacher was unable to talk about specific problems a given pupil might have.

The reason for discussing tests with pupils individually in the experimental group was due to the following two situations: first, not all students who wrote the same test wrote it at the same time and second, different achievement levels required totally different tests.

Managerial tasks (4.0)

In each classroom a number of management tasks have to be performed in the course of a lesson. The graph on page 86 shows the differences and similarities between the experimental groups and

TESTING

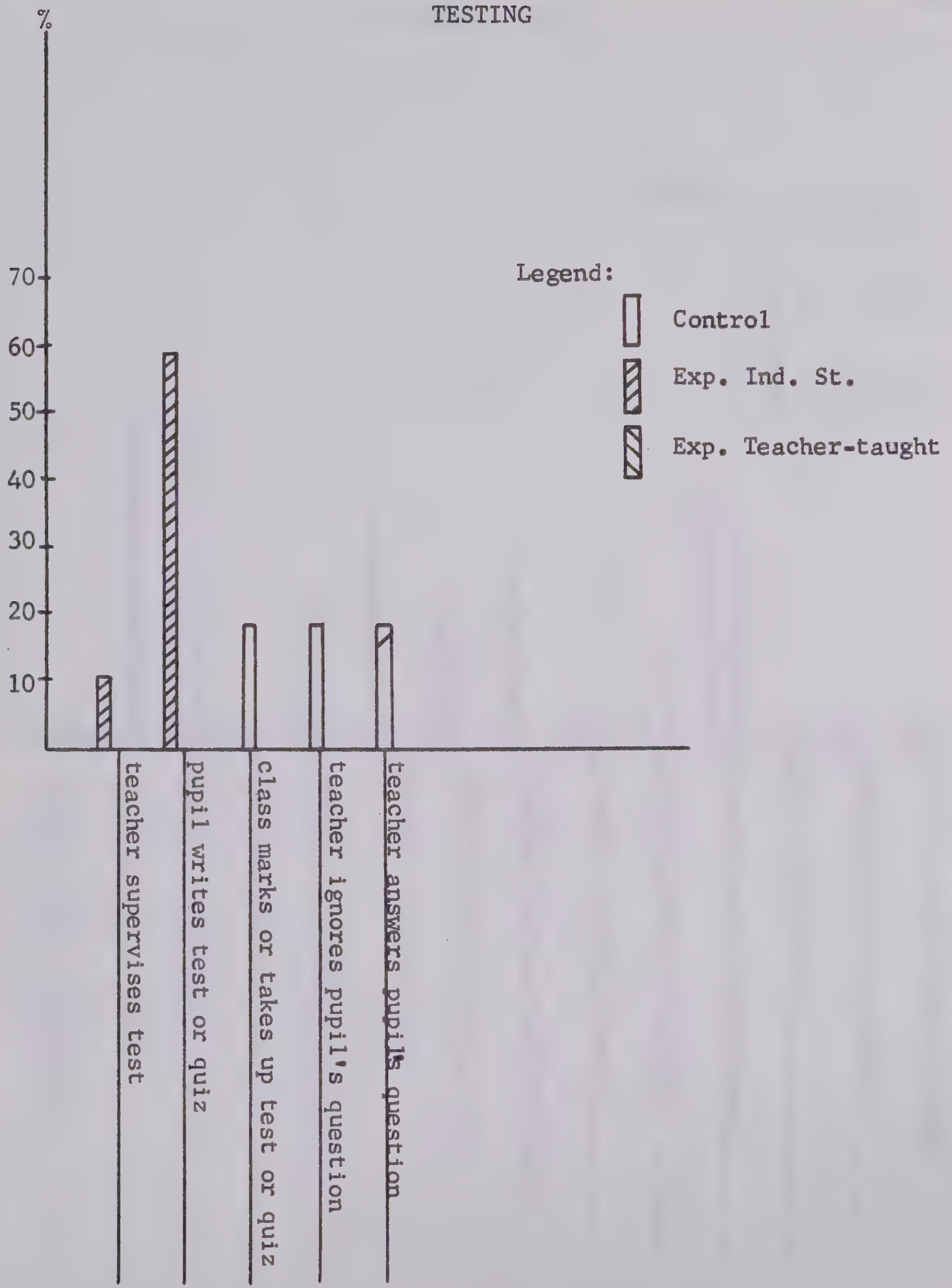


FIGURE XXI

GRAPH SHOWING PERCENTAGE DISTRIBUTIONS FOR

BEHAVIORS RELATED TO TESTING

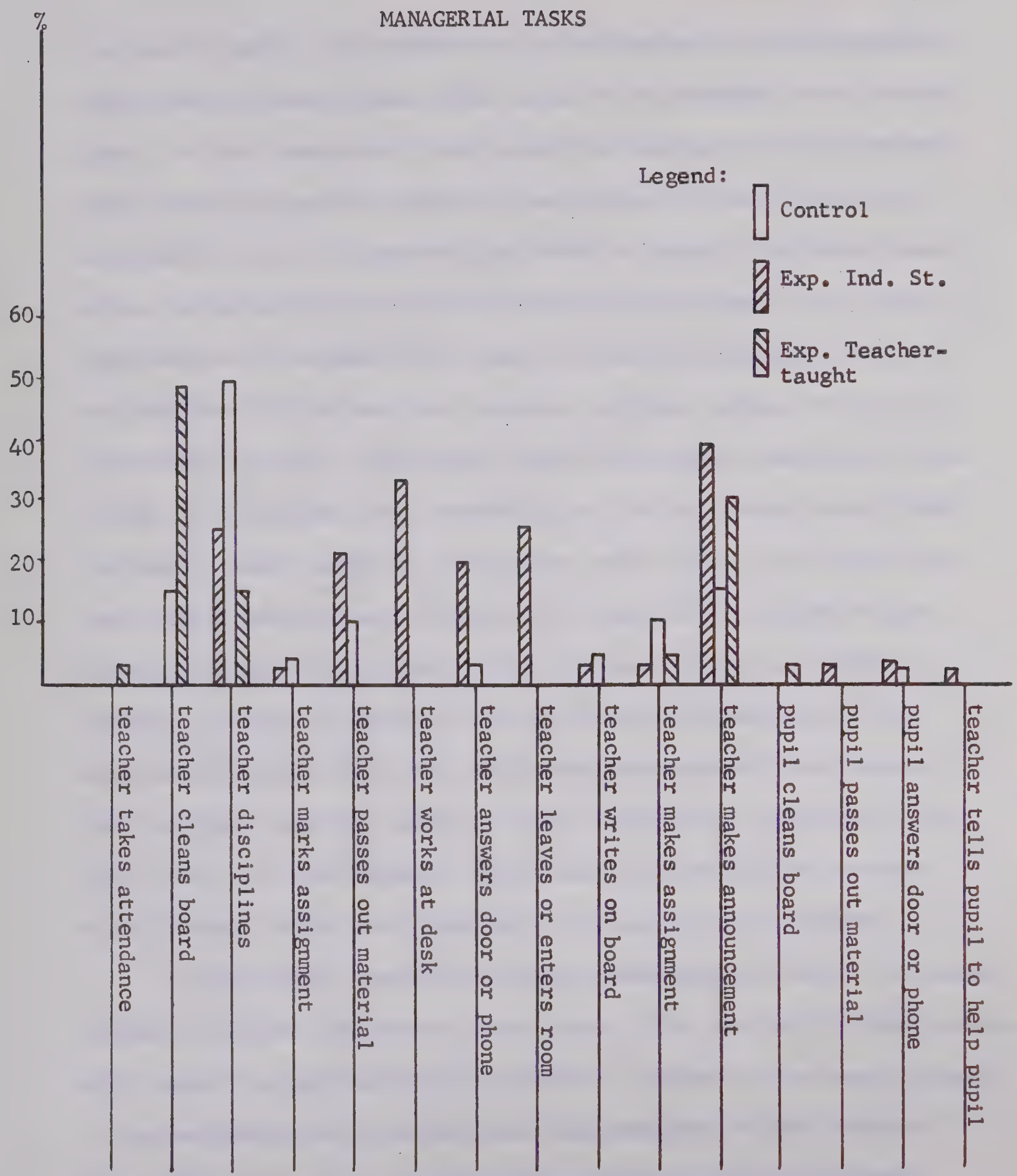
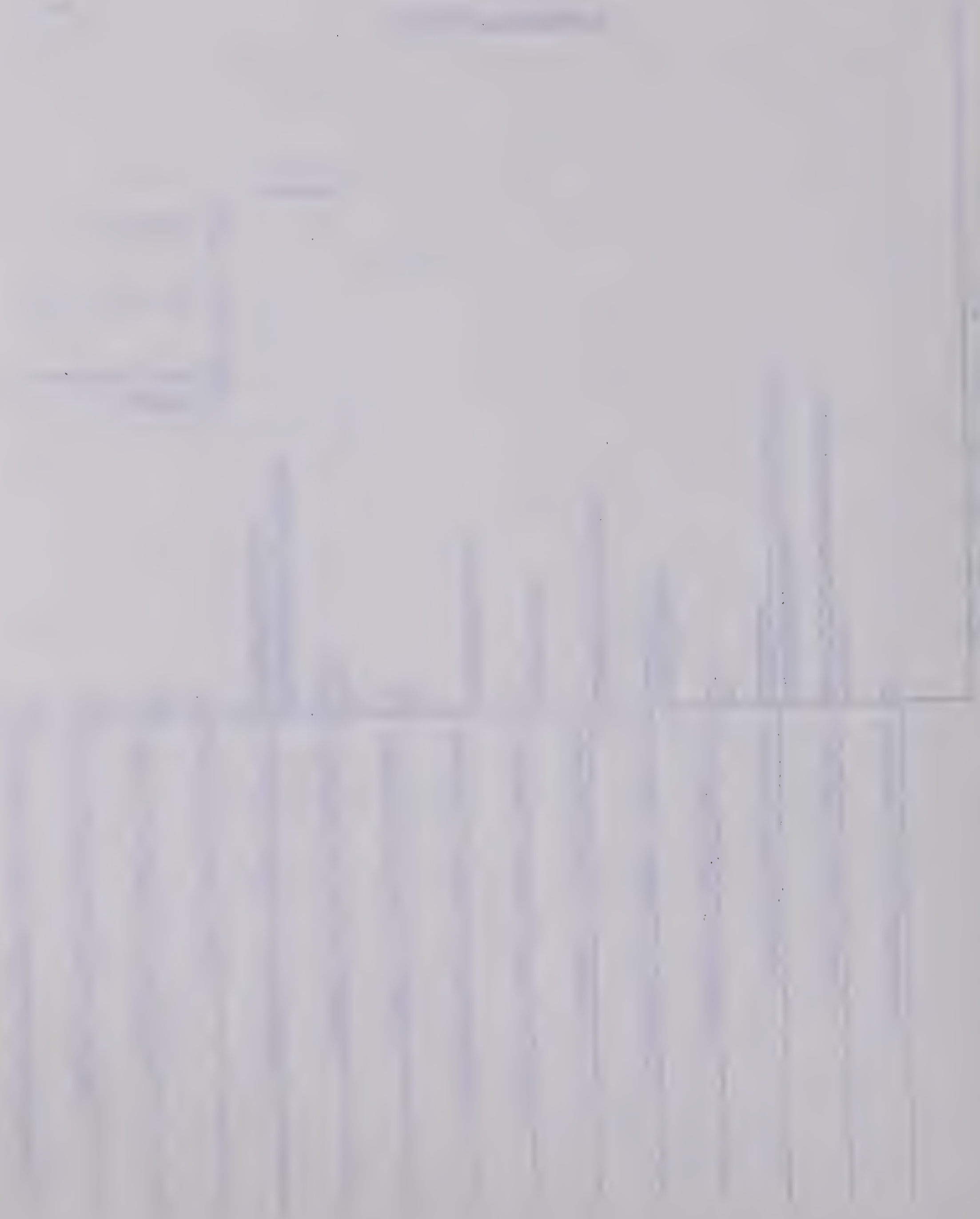


FIGURE XXII

GRAPH SHOWING PERCENTAGE DISTRIBUTIONS FOR

BEHAVIORS RELATED TO MANAGERIAL TASKS



the control group. The activities of the teacher in the independent study group differed greatly from those of the teacher in the control group. In the independent study group the teacher could be observed quite often passing out materials, working at his desk, leaving or entering the room, or answering the door or phone. Upon close examination the writer found that the teacher could be seen at his desk more frequently, because pupils kept him there by coming to him, as was mentioned in the seat work section, and also because he had more record-keeping to do. The writer found the teachers worked as a team. If one of the teachers was conducting the teacher-taught class, then the others looked after his independent study class. This explained the frequent entering and leaving of a room, for the teacher-taught classes were conducted almost daily, but every day by a different teacher. It must be noted at this point that the teachers in the experimental group taught the grade seven mathematics simultaneously. The classrooms were too small to take a part of the pupils that were left alone in a room whenever their teacher conducted the teacher-taught class. Hence the situation of a room without a teacher.

The teachers were also observed answering the door or the phone frequently in the independent study group. This was due to consultations with respect to materials and procedures. Because of the team approach it was necessary that everyone knew what happened in the different rooms. Thus if an error was found the others had to be informed. Since the materials were used for the first time and unavoidable errors in materials and procedures did occur, it was necessary to rectify these immediately in every room.

Since pupils worked at their own rate it was possible to see a teacher hand out some new packets of materials almost every class period. The number of announcements was greater in the experimental setting because of the nature of the program. A greater number of variations were possible in the experimental setting and generally many of the announcements were either to inform the pupils about changes in the materials or procedures or reminders to the pupils of things to do and to watch for.

The cleaning of the board by the teacher was observed only in those settings where formal teaching occurred and was never observed in the independent study setting.

Disciplining of a class as a whole occurred least often in the experimental teacher-taught class and most frequently in the control classes. As a matter of fact the teachers of the control classes were observed disciplining the entire class about twice as often as were the teachers in the independent study setting. This was pointed out earlier in the classroom climate section. The writer has no particular explanation for this except that maybe the teachers in the control group had more difficult classes to control. Otherwise it might have to be concluded that the experimental program fostered a better attitude towards the learning of mathematics.

A number of behaviors were observed only occasionally as the graph shows and thus it is not possible to say too much about them other than that these were observed and did occur sometimes in one setting and not in another setting.

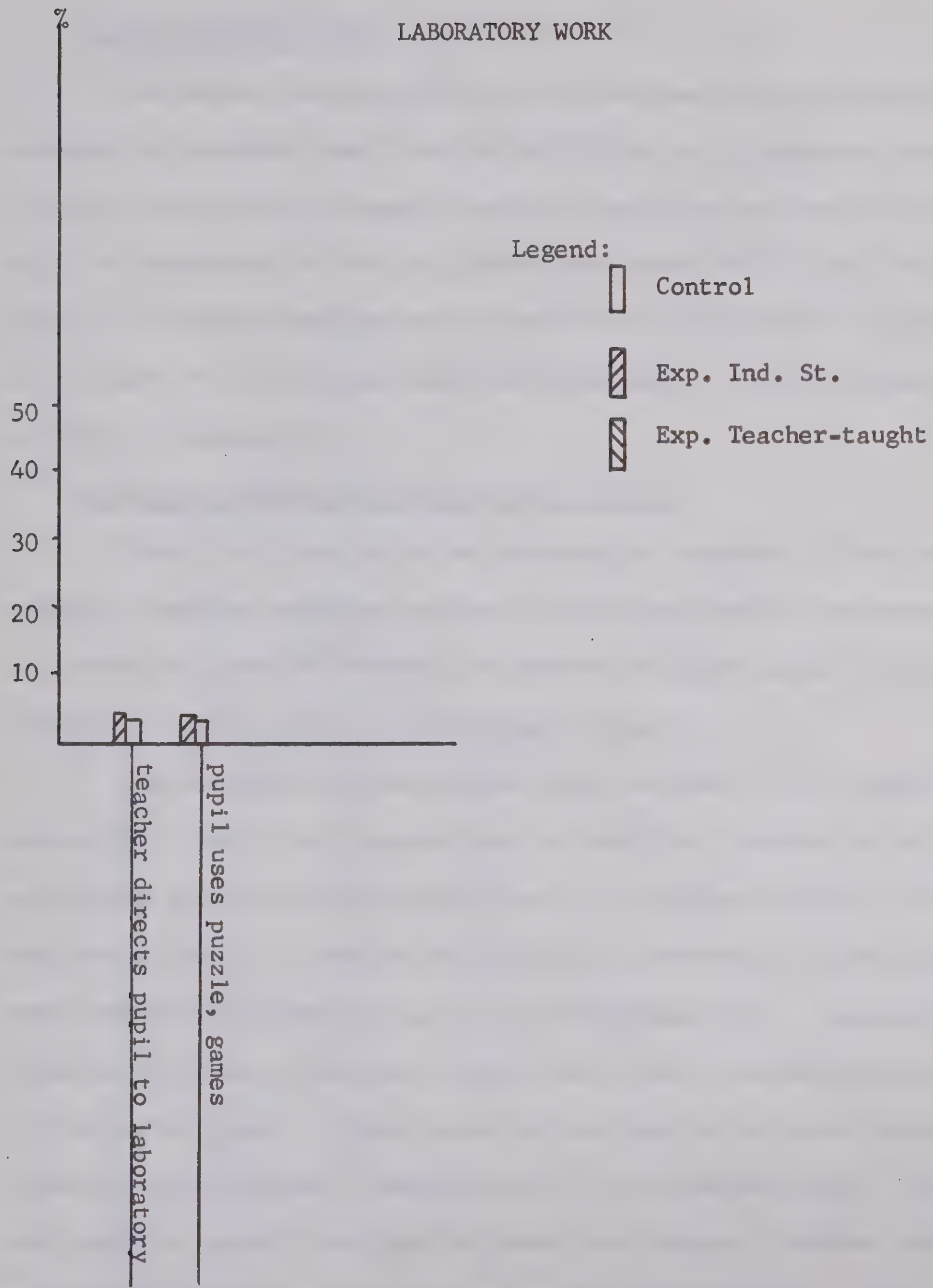


FIGURE XXIII

GRAPH SHOWING PERCENTAGE DISTRIBUTIONS FOR

BEHAVIORS RELATED TO LABORATORY WORK

Laboratory Work (7.0)

The writer included this section because the experimental program had provided some time for activities of a laboratory nature. The writer was able to observe the use of puzzles and games by pupils on a few occasions in both the independent study setting and the control group. In neither setting was the use of this approach a very significant part of the regular classroom procedure. The graph referring to this is on page 89.

Affective Teacher Talk and Actions (10.0)

This final section on the Observation Schedule differs in its analysis from the previous sections in that the results are reported as frequencies per 100 minutes per teacher for each group. The 100 minute period was chosen to make graphing easier.

The teacher talk and actions were analyzed in two separate categories. The first category was the talk that related to the class as a group and the second category was the talk that related to the individual pupil. It was often difficult to determine if the teachers were addressing the whole class or an individual pupil. Generally speaking, in those situations, the writer noted it as being directed to the entire class. By and large the teachers in a formal teaching situation directed their remarks mainly to the entire class. The two graphs on page 91 and page 94 show this clearly. Another problem encountered in the analysis of the teacher talk occurred in the independent study situation where the teacher could not be heard by the observer whenever the teacher was dealing with individual pupils. The entries for the talk directed to individual pupils should have

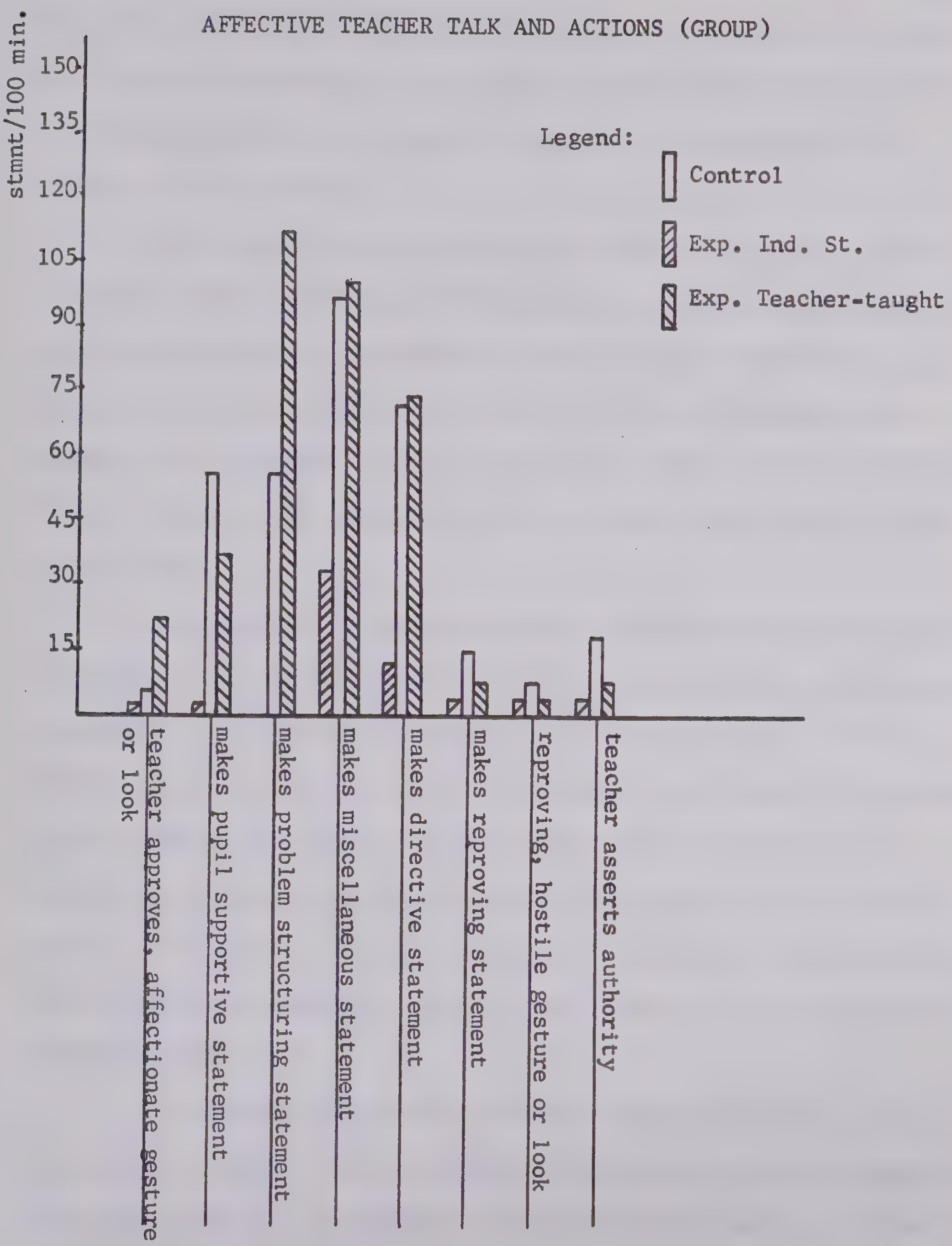


FIGURE XXIV

GRAPH SHOWING PERCENTAGE DISTRIBUTIONS FOR BEHAVIORS RELATED TO AFFECTIVE TEACHER TALK AND ACTIONS FOR THE GROUP

been many times greater for the independent study group, but it was not possible to record it. The graph on page 91 shows the analysis of the statements and actions that were made on the average per teacher per 100 minutes.

The teachers in the independent study classes were observed to address their classes very infrequently. The only significant verbal behavior measured was the making of miscellaneous statements. The reason for this is simple. The teachers in the independent study setting only addressed the class as a whole when a general announcement had to be made. The statements never related to any specific mathematics task.

The teachers in the experimental teacher-taught class and the teachers in the control class were alike with respect to miscellaneous statements and directive statements. In both cases they differed markedly from the teacher in the independent study class. The teachers in the experimental teacher-taught class made significantly more problem-structuring statements than did the teachers in the control group. On the other hand the teachers in the control group made more pupil-supportive statements than did the teachers in the experimental teacher-taught class.

The teachers in the experimental teacher-taught class were much more positive in their actions and looks than they were negative. The graph (page 91) illustrates this distinction clearly. It also illustrates how the teachers in the control group are much more negative in their actions than positive. The results mentioned in this paragraph seem to fall in line with what was found in the class-

room climate section with respect to teacher affection for his pupils.

As was noted at the outset of this section, there should have been a much higher entry for the independent study setting, but this was not possible for the observer could not hear what the teacher said in most cases when the latter was dealing with individual pupils. Only the teacher's actions and gestures could be recorded and hence a distorted picture was obtained. Yet something interesting appears when the independent study group, the experimental teacher-taught class group, and the control group are compared. It appears that the teachers in the experimental teacher-taught class setting and the teachers in the control setting directed very little attention to the individual. In almost all categories the independent study group scored higher or as high as did the other two groups even though the writer experienced extreme difficulty in scoring the former group.

A glance at the graph on page 94 will confirm what the writer experienced in his attempt to score this section of his observation schedule. At the same time the reader will note that the actions on the part of the independent study group were significantly more positive than negative. The latter corroborates what was found in relation to the teacher talk and actions as directed to the whole group in the experimental teacher-taught class. It is not surprising however to find this for the teachers in the independent study group and the experimental teacher-taught class group were the same teachers.

With this section the writer has completed the analysis of the findings of the observation schedule. The following section deals with the analysis of the data from the log sheet.

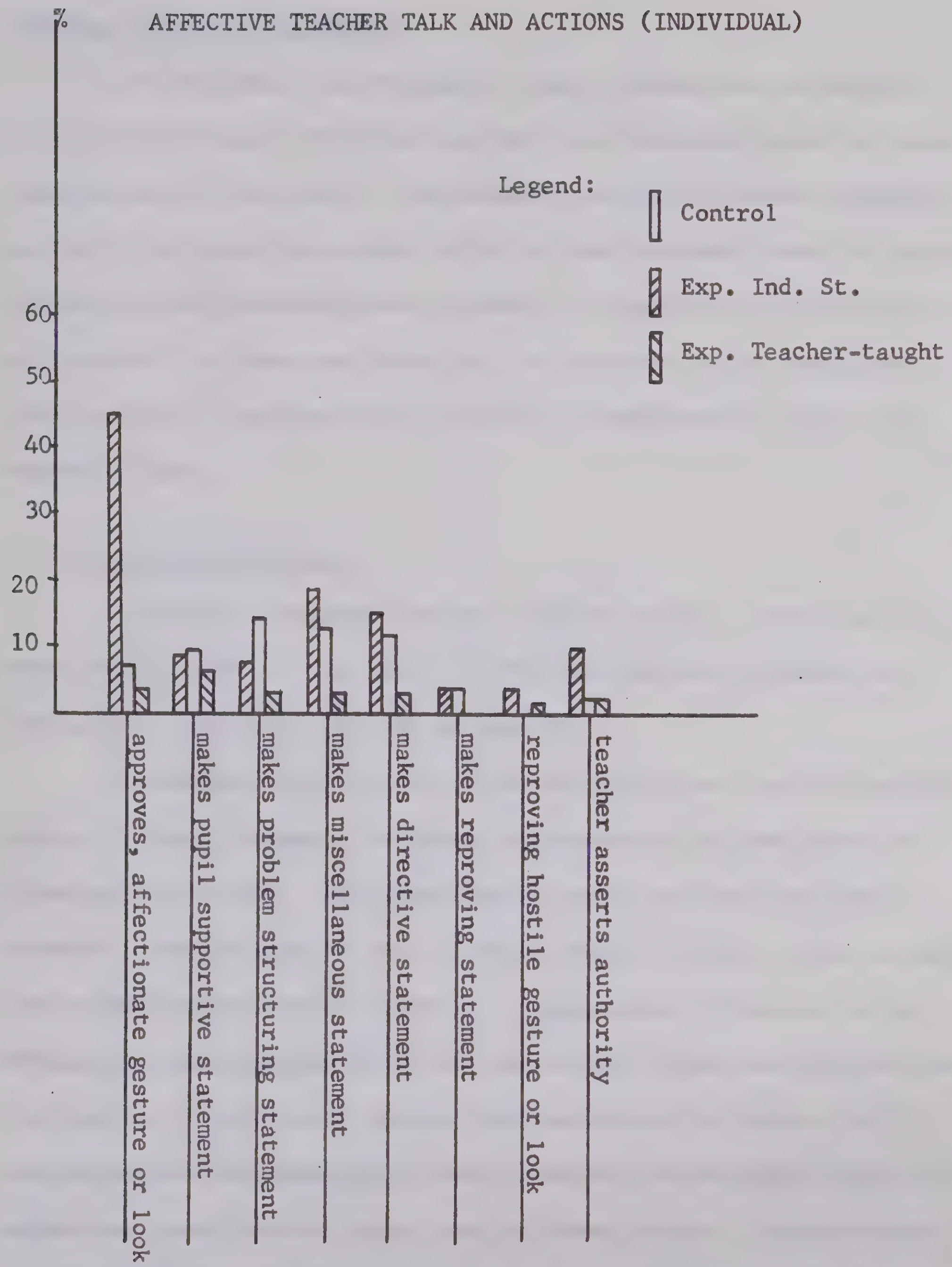


FIGURE XXV

GRAPH SHOWING PERCENTAGE DISTRIBUTIONS FOR BEHAVIORS RELATED TO AFFECTIVE TEACHER TALK AND ACTIONS FOR THE INDIVIDUAL

Findings From the Log Sheet

The log sheet was designed to gain information in relation to two major areas. The first was the time distribution of different teaching activities used by the teachers during the normal teaching period. The second area dealt with the time teachers spent on tasks related to their teaching task outside of class time. The findings with respect to those two areas will be reported in the remainder of this section. Sometimes pupil behavior is mentioned to imply the teacher's task.

Classroom Activities

In order to compare the two teaching styles a graph was prepared with respect to the eight activities that were examined on the survey. The graph follows on page 96.

It becomes quite evident from the graph that the two teaching styles do indeed demand a different distribution of time spent on classroom activities. The experimental group reported that their students were assigned to work at their desks for about twice as much time as the control group reported. Approximately the same ratio between the experimental group and the control group was reported for the helping of individual pupils, the teaching of different ability groupings, and the teaching of small groups. In the latter case the writer had specified the group size as three to six. The reporting of the teachers in the experimental group with respect to the above activities shows a good deal of consistency, for the total time devoted to the helping of individual pupils, the teaching of different ability groups and the teaching of small groups should be roughly equivalent

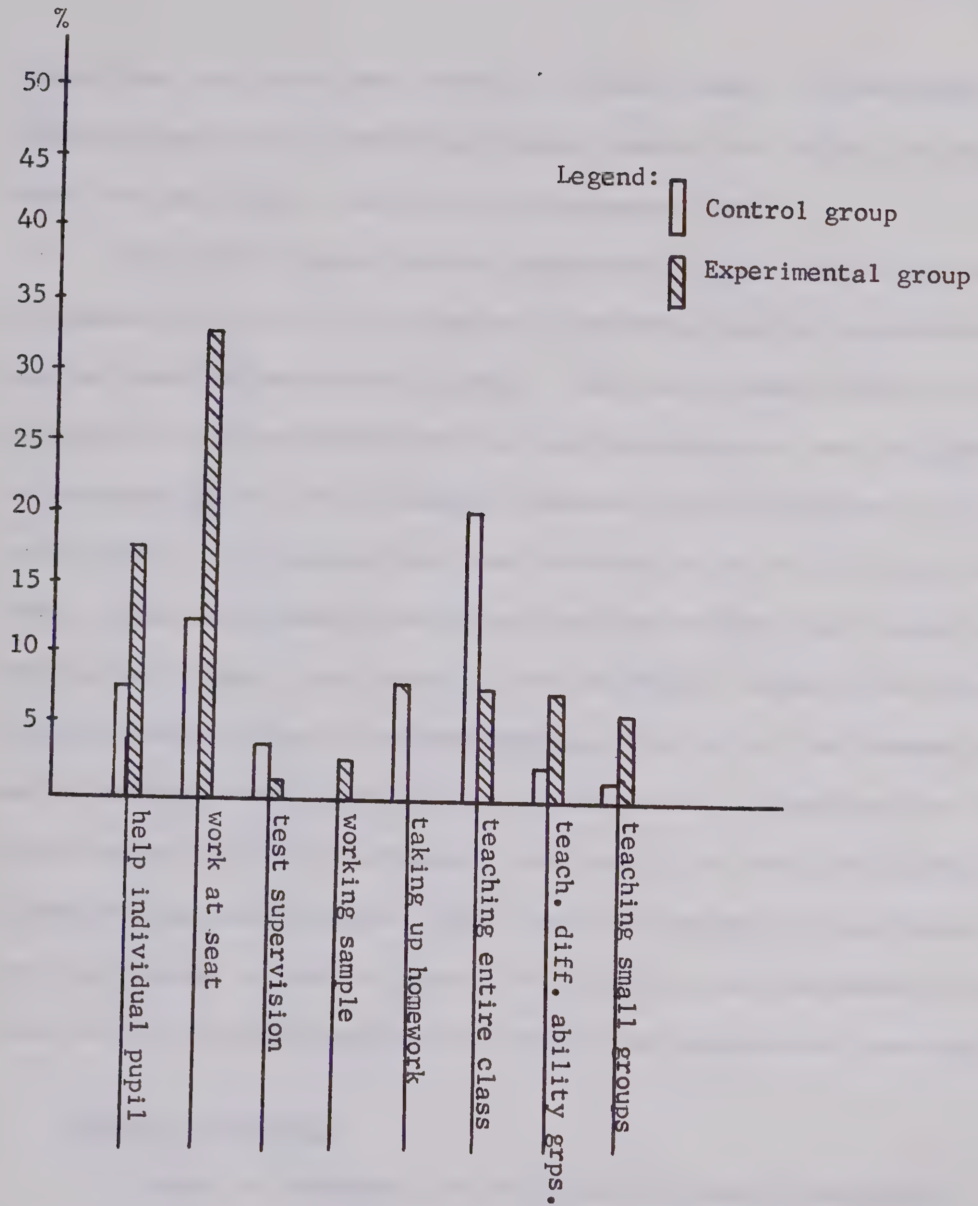


FIGURE XXVI

CLASSROOM ACTIVITIES AND THEIR AVERAGE PERCENTAGE DISTRIBUTION

to the time the pupils were working at their desks. If the reader consults the graph it shows that these times do not differ too greatly. About the same holds true for the experimental group.

The control group devoted considerably more time to the teaching of the entire class, the taking up of homework, and the supervision of testing than the experimental group. This was to some extent to be expected for the individualized instruction materials were designed to eliminate chores like taking up homework and the teaching of the entire class. The reason that some teaching of the entire class was shown by the experimental group was that the teachers who taught in the teacher-taught class aspect of the experiment reported the time divisions used in that classroom setting. The writer has however no way of telling which times relate to that aspect. Therefore the time distribution for the experimental group would, in fact, be slightly different than was reported. The times for the experimental group were an average of the two aspects of teaching, the independent study mode and the teacher-taught mode, used in the experimental setting.

Related Activities

A graph was prepared for the purpose of easier comparison between the experimental group and the control group with respect to the times used for activities related to the teaching task outside of class time. From the graph some significant differences appear between the two groups with respect to time spent on marking tests, record-keeping, departmental meetings, and group planning. The writer examined these results just a little more closely to see if the experimental program would always require this much time in the above areas.

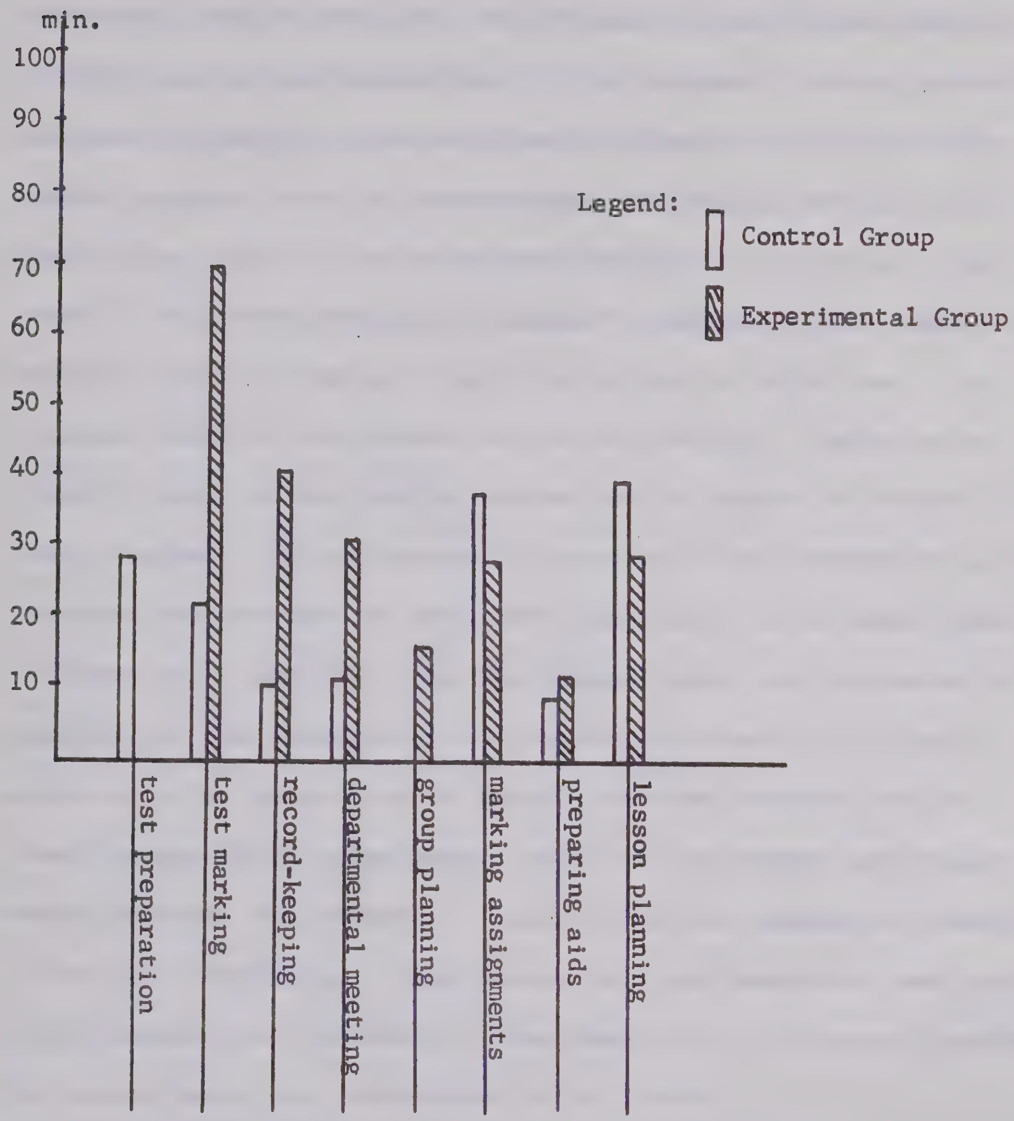
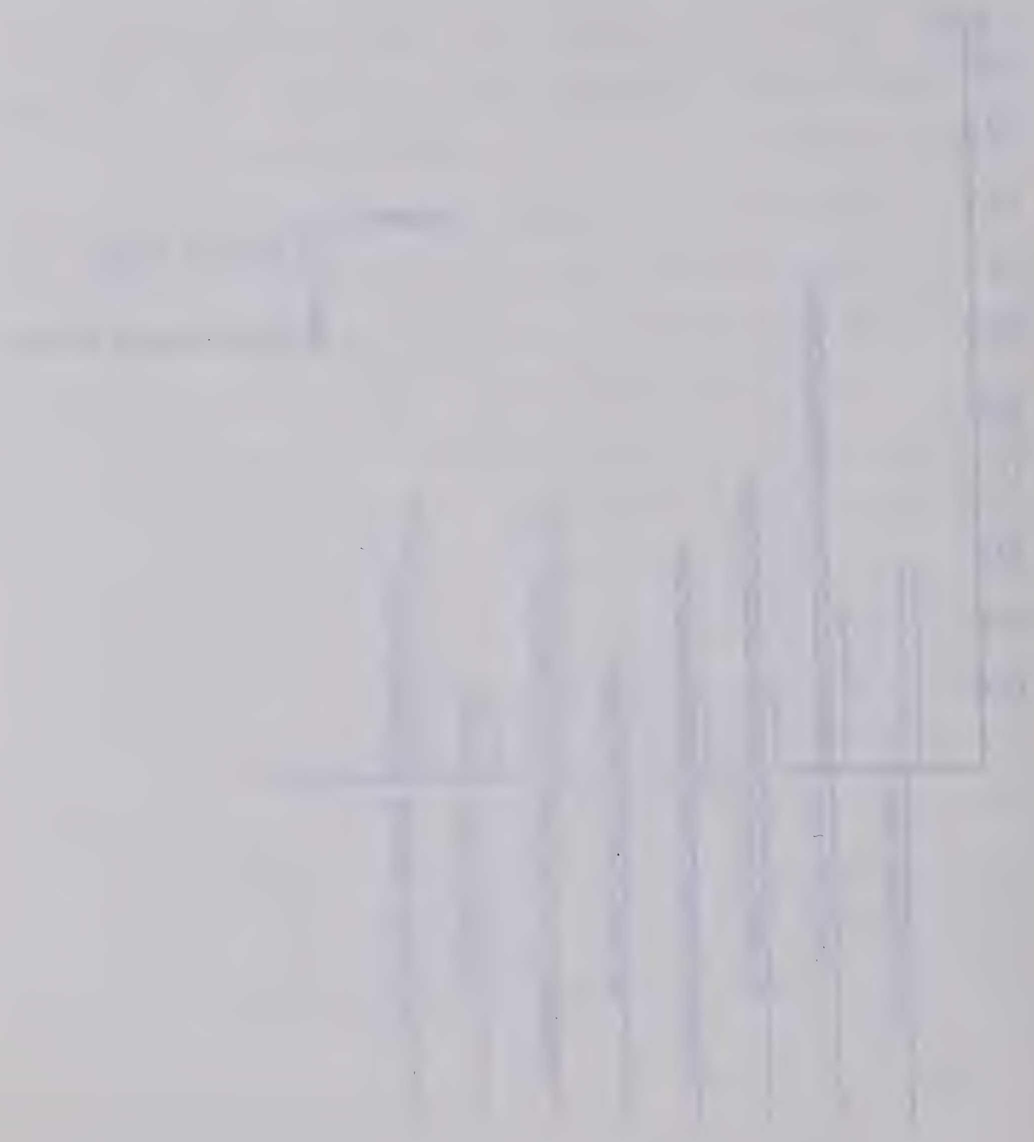


FIGURE XXVII

AVERAGE WEEKLY PREPARATION AND MARKING TIMES PER TEACHER



The marking load of tests will not diminish as the program continues, for the tests are an integral part of the program. Testing occurred much more frequently in the experimental program than it did in the control program. Many of the tests took the pupils more than one 45 minute class period to write for every objective studied had to be tested. The marking required a teacher's judgment in many cases to decide if the pupil had met the criterion for the objective. Thus a teacher aide in this program was not too helpful. Another area directly linked to the testing program was the keeping of records on pupil progress. The achievement of every pupil with respect to each objective was recorded for each individual pupil on his record sheet. This work could have been done by a teacher aide, but the teacher aide available to the teachers did not have any time left to do this in addition to the supervision of testing and other clerical duties. Then, because of the experimental nature of the program many departmental meetings were required. Coupled with this program of planning is the actual teaching. Since the teaching was based on a team approach, group planning was a necessity. The time required for group planning the writer cannot see diminishing in the future.

The experimental group required about the same amount of time as the control group for such areas as marking of assignments, the preparing of aids, and the planning of lessons by each individual teacher. In the area of test preparation the experimental group did not expend any time at all, for the tests were prepared and included with the material. As was stated before, the tests were an integral part of the material.

What do the results of this section indicate? These simply point out one significant factor and that is the teacher in the experimental program spends about seventy minutes per week more on activities related to this program than his counterpart does on the control program.

With the above remarks the writer concludes the analysis of the data obtained from the log sheet.

Findings From the Questionnaire

The questionnaire was designed to gather information about areas which the previous two instruments had left untouched, i.e. how did the teachers plan? how did the teachers feel the program affected the pupils? and what were their feelings about the program? The first part of the questionnaire was designed to make comparisons between the experimental group and the control group. Hence the first 32 questions were given to both groups. Questions 33 to 55 were given to the experimental group only. Usually more than one question referred to the same concept and hence the replies to those questions will be grouped for the consideration of the results. In order to facilitate the discussion the section will be divided into a number of subsections each dealing with a particular set of questions. The questions on the questionnaire to which the subsection relates will be indicated following its heading.

Planning the Unit (Questions 1 to 6)

The experimental group indicated that they planned each unit of work together as a department. Following that they had a committee

of teachers to prepare the necessary materials to be used for the particular unit. The control group indicated that each teacher planned each unit of work by himself and that each teacher prepared his own materials.

Planning of Lessons (Questions 7 to 10)

The experimental group indicated a number of approaches. The majority showed that they planned each lesson alone and not as a department. Two of the teachers seemed to plan together by the indications on their questionnaires. Some of the teachers felt that the lessons were planned by the department as a whole. The control group showed again that each teacher planned the lessons by himself. Neither of the two groups reported that the lessons were planned in cooperation with the pupils.

Teaching of Each Class (Questions 11 to 14)

This section deals with the manner in which teachers dealt with their classes during the course of a classroom period. The experimental group indicated that each teacher teaches his assigned class for most of each period, however there was also an indication that they assisted each other within the department in the teaching of the classes. The term teaching as used by the experimental group has to be viewed in a different light than traditionally is the case. The teacher in the experimental setting acted as a resource person and a diagnostician who mainly dealt with the pupils in his class on a one-to-one basis. The teacher was available to any pupil in his class during the entire class period. The control group indicated that

each teacher looked after his assigned class for the entire period and that there was no assistance from other teachers within the department.

Teaching Method Used (Questions 15 to 19)

The manner in which individual lessons were taught differed considerably between the two groups. The experimental group indicated that they used: ability grouping, the teaching of small groups, independent study with prepared materials, and whole class instruction. The latter differed from the regular classroom instruction in that the pupils who were taught as a whole class were actually attending a lecture to supplement their independent study. The control group indicated that they taught the entire class at once. The control group also indicated that some small group work using prepared materials had been employed. The differences between the two groups would seem to indicate that the individualized study approach promoted the greater variety of approaches.

Teacher Expectations (Questions 20 to 22)

The experimental group reported they did not expect all their pupils to progress at the same rate. Neither did the experimental group expect pupils of the same ability to progress at the same rate. They indicated they expected each pupil to progress at his own rate. The control group also indicated they did not expect all their pupils to progress at the same rate. They did expect pupils of the same ability to progress at the same rate. The control group did not allow pupils to progress at their own rate. There was thus a significant

difference between the two groups in expectation with respect to a pupil's progress. The experimental group expected each pupil to progress at his own rate and the control group expected pupils of the same ability to progress at the same rate.

Teaching Style (Questions 23 to 25)

The experimental group reported that they taught as a team and that they had the use of a teacher aide. The control group on the other hand reported that they taught independent of each other and that they did not have the assistance of the teacher aide.

The Teacher Aide (Questions 26 to 32)

From the answers given it appeared that the teachers in the experimental group had the assistance of a teacher aide for more than two days per week. The writer found his question 26 lacking and checked afterwards with the teachers on this one point and found the teachers as a group had the assistance of one teacher aide for five half days per week. The teacher aide assisted the teachers with supervision of testing, she did some marking of tests for a couple of teachers, while others had her assist them with the preparation of teaching aids and apparatus. The teacher aide did not assist the teachers in the keeping of records or the supervision of classes. The teachers however had the teacher aide assist them with clerical duties such as collating, duplicating and distributing materials.

With the above subsection that part of the questionnaire which dealt with matters relating to both the experimental group and the control group is concluded. The remaining part of the questionnaire

applied to the experimental group only and reflected mainly the feelings and reactions the teachers had about the program.

Reactions to the Experimental Method (Questions 33 to 55)

In this section the teachers were asked to compare the experimental method with their regular classroom experiences. From the answers the teachers gave on the questionnaire the following results were drawn.

The teachers found the experimental method aided them in getting to know their pupils better. They felt they were able to discover weaknesses of the pupils faster in the experimental setting. These two findings would indicate that pupils could be helped sooner to overcome their difficulties and the teachers would be able to start remedial work at an earlier stage.

All teachers reported the experiences with the experimental method had influenced their regular classroom teaching. The teachers found they used a fair number of the ideas developed for the experimental method in their regular classroom teaching which indicates the teachers' regular classroom methodology had been affected.

All the teachers reported they had become more aware of individual differences since becoming involved in individualized instruction. As a consequence almost all of the teachers were trying to do more for the individual pupil in the regular classroom which seems again an indication of how the teachers felt the experimental program was affecting their regular teaching behavior. The foregoing reactions indicate a very positive feeling of the teachers about the experimental program. Therefore it is not surprising that all but one teacher indicated they had a definite preference for individualized

instruction over the regular teacher taught class.

The teachers reported that the experimental method permitted them to do more for the individual pupil; that they could do more for small groups of pupils who had common difficulties; and that they even were able to do more for a class as a whole. These findings were of course based upon the opinions of the teachers involved and hence are completely subjective. However in view of the heavy workload involved as pointed out in the discussion of the results on the log sheet, the above reactions are more commendable.

What was the teachers' feeling about the affect the program had on the pupils? Once again a very positive result. Most of the teachers felt the experimental program did more for the above average and average pupil than did the regular teaching method. The majority even felt that the below average pupil gained more from the individualized program. The majority of the teachers reported also that individualized instruction did not slow the progress of the above average, average, and below average pupils. It must be reported here that the latter finding is not unanimously reported by the teachers. Some of the teachers felt the experimental method slowed the progress of the pupils.

A noteworthy finding was related to discipline. None of the teachers reported that they found it easier to maintain discipline in the experimental class as compared to the regular class. The majority claimed it was more difficult to maintain discipline in the individualized classroom and two reported they found no difference between the discipline in the experimental setting and the regular setting.

The last two questions on the questionnaire were open-ended questions where the teachers were asked to list their likes and dislikes. The writer will report the dislikes of the teachers with respect to the experimental program first, then the likes.

Dislikes

The teachers disliked the excessive marking and record-keeping required by the program. Coupled with this was the great amount of time required for planning, i.e. departmental planning, team planning and individual lesson planning. It was not surprising to find these comments, for from the log sheet it became apparent that in these areas the teacher had to spend much more time than normally was required.

Another area of dislikes was with respect to the work habits of the pupils. The teachers found the program tended to promote poor work habits in a number of pupils. They also found that the immature student took advantage of the situation and did as little work as possible. The immature pupil with poor work habits swayed easily those who seemed to have a little more responsibility. Coupled with the above complaint was the finding that many important skills were lost such as doing assignments well, neatness, and use of certain materials. The writer does not know what materials the teachers had in mind when they wrote this.

The teachers disliked the noise level with which they had to work. This is an aspect of the program which cannot completely be controlled for pupil interaction was a part of the design of the program. Finally the teachers felt there was a lack of a general teacher-class rapport. The writer does not fully grasp the meaning of the

latter comment, for in section 10 of the observation schedule greater communication between the teacher and the pupil was shown. It is possible that the teachers were referring to the fact that pupils were losing the ability to receive instruction from a teacher in a class situation.

Likes

The teachers liked the way in which the program facilitated pupil assessment. The teachers reported they had instant assessment of a pupil's progress which was supplied by the record page. They also stated they could thus identify the slow pupil quickly. The program facilitated the chance of getting to know the pupil and thus the teachers were able to evaluate the pupil more accurately. As one teacher put it: "It makes me more aware of the individual abilities, attitudes and interests of students within a given class." Another area of likes is the way the experimental program facilitated pupil progress. They liked the program because it did not hold up the fast student so he was able to move ahead and get some enrichment. The slow student also benefitted for he was able to get more help. Another area the teachers liked was the one of involvement. First, pupils were more involved in the teaching of the subject, for many acted as student helpers. Secondly, the team approach to the program actively involved the teachers which the teachers felt was a great advantage. The team approach made it possible to learn from others and to share ideas and burdens with others. Another reason for liking the program was that it promoted a development of responsibility in many pupils.

Finally the teachers liked the program in general for as they stated it "turned on" more pupils to mathematics than traditionally, which made for a better (happier) relationship between teacher and pupils.

Summary and Conclusions

The writer set out to answer a number of questions. He is now in a position to answer these questions, at least in part. In order to facilitate the making of the summary the writer has prepared a profile based on the observation schedule and the log sheet for the experimental groups and the control group. The profiles appear on the next pages.

To answer the question "What does the teacher do in the individualized instruction setting?", the writer has to point out that the answer has to be given in two parts. The first part deals with the teacher in the independent study class and the second part deals with the teacher in the teacher-taught class.

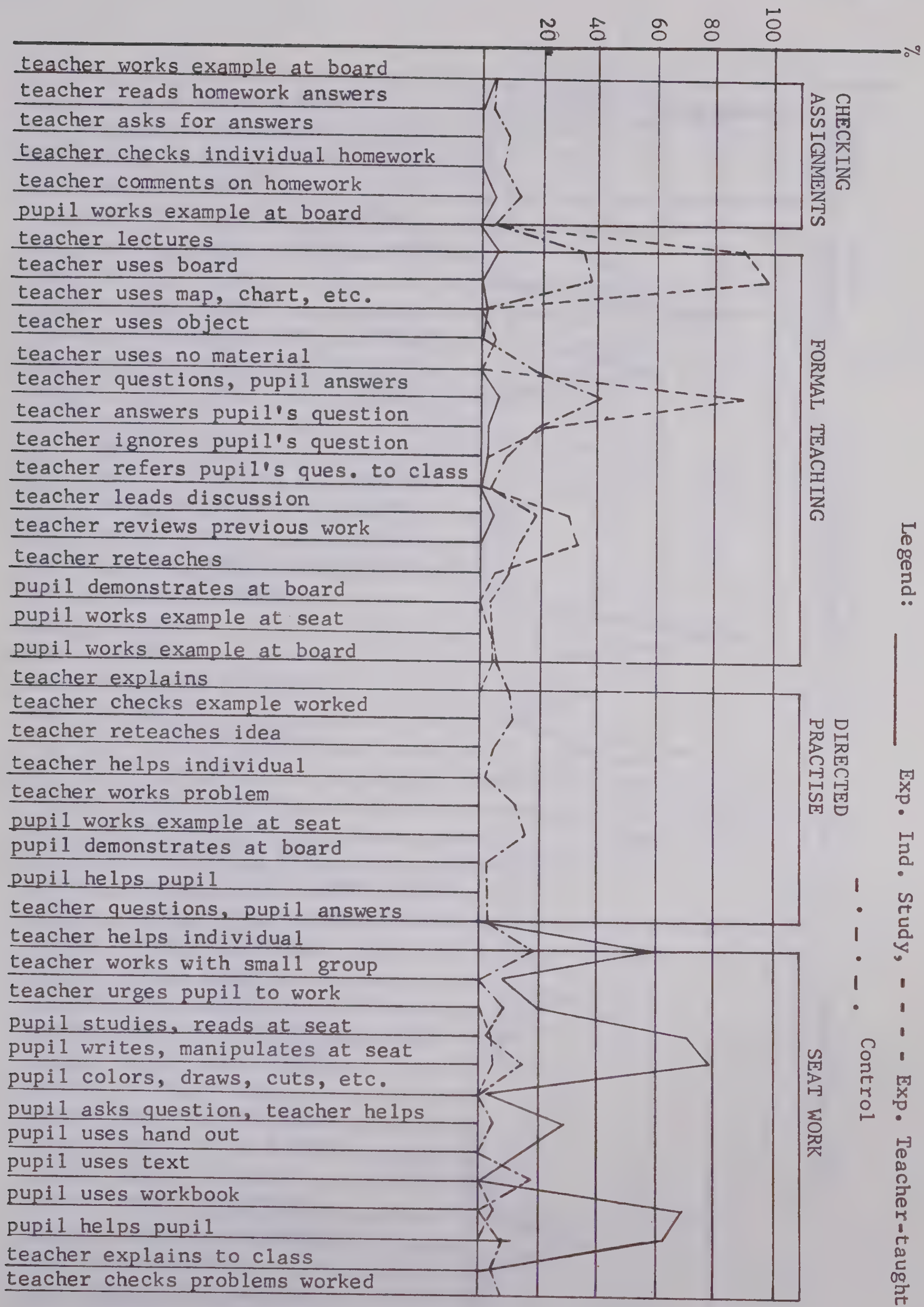


FIGURE XXVIII

PROFILE BASED ON OBSERVATION SCHEDULE

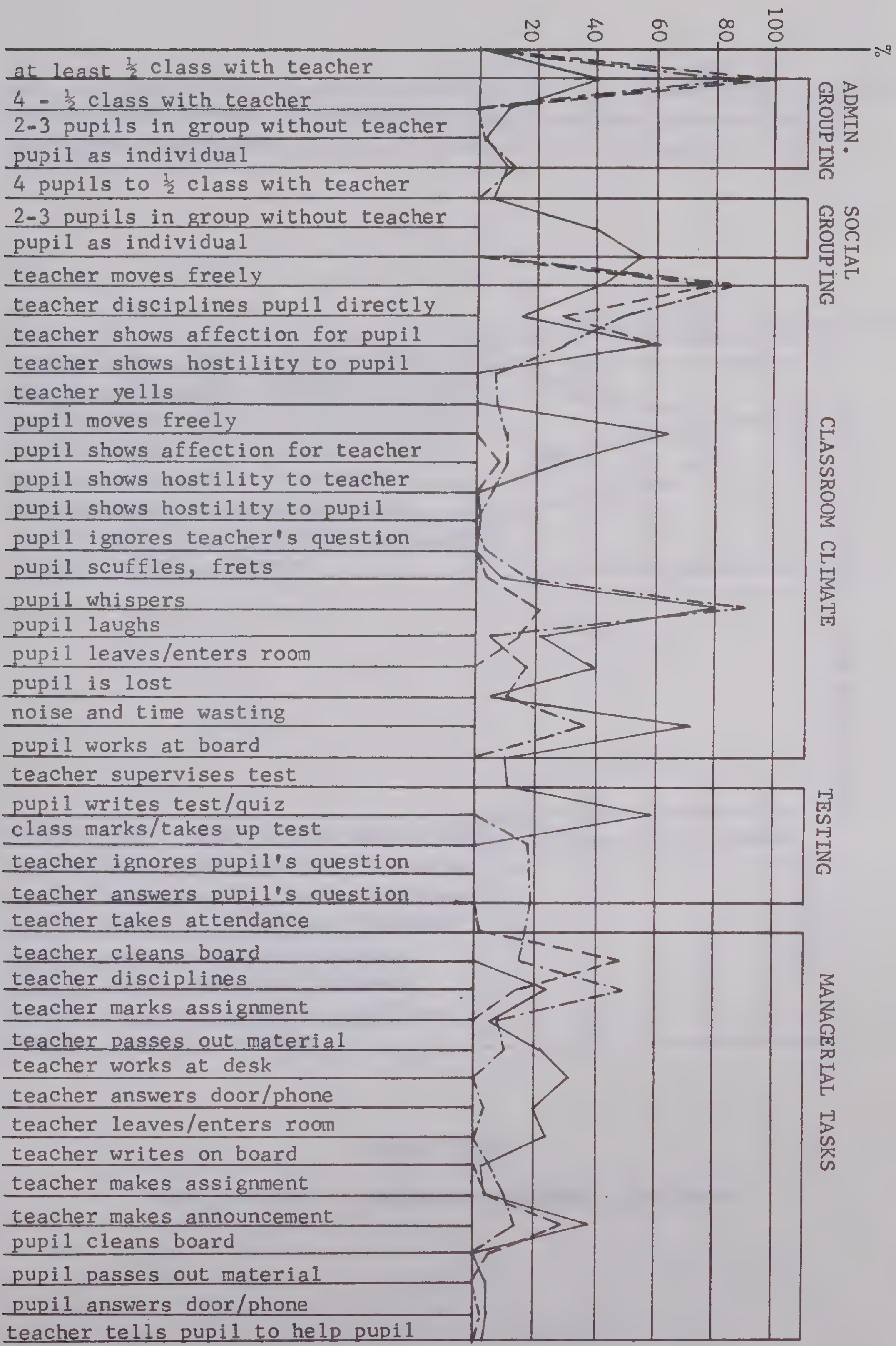


FIGURE XXVIII

PROFILE BASED ON OBSERVATION SCHEDULE CONTINUED

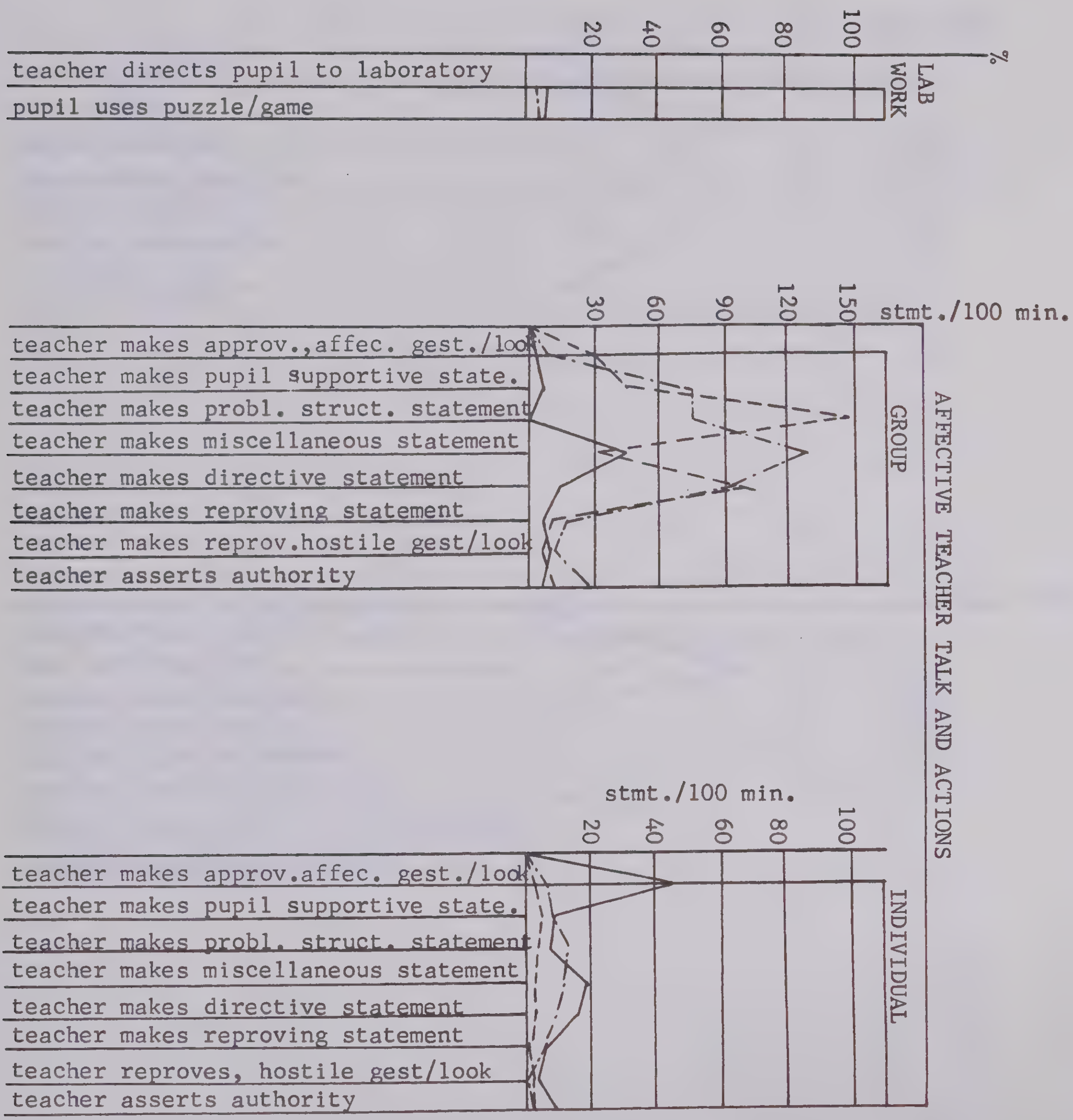


FIGURE XXVIII

PROFILE BASED ON OBSERVATION SCHEDULE CONTINUED

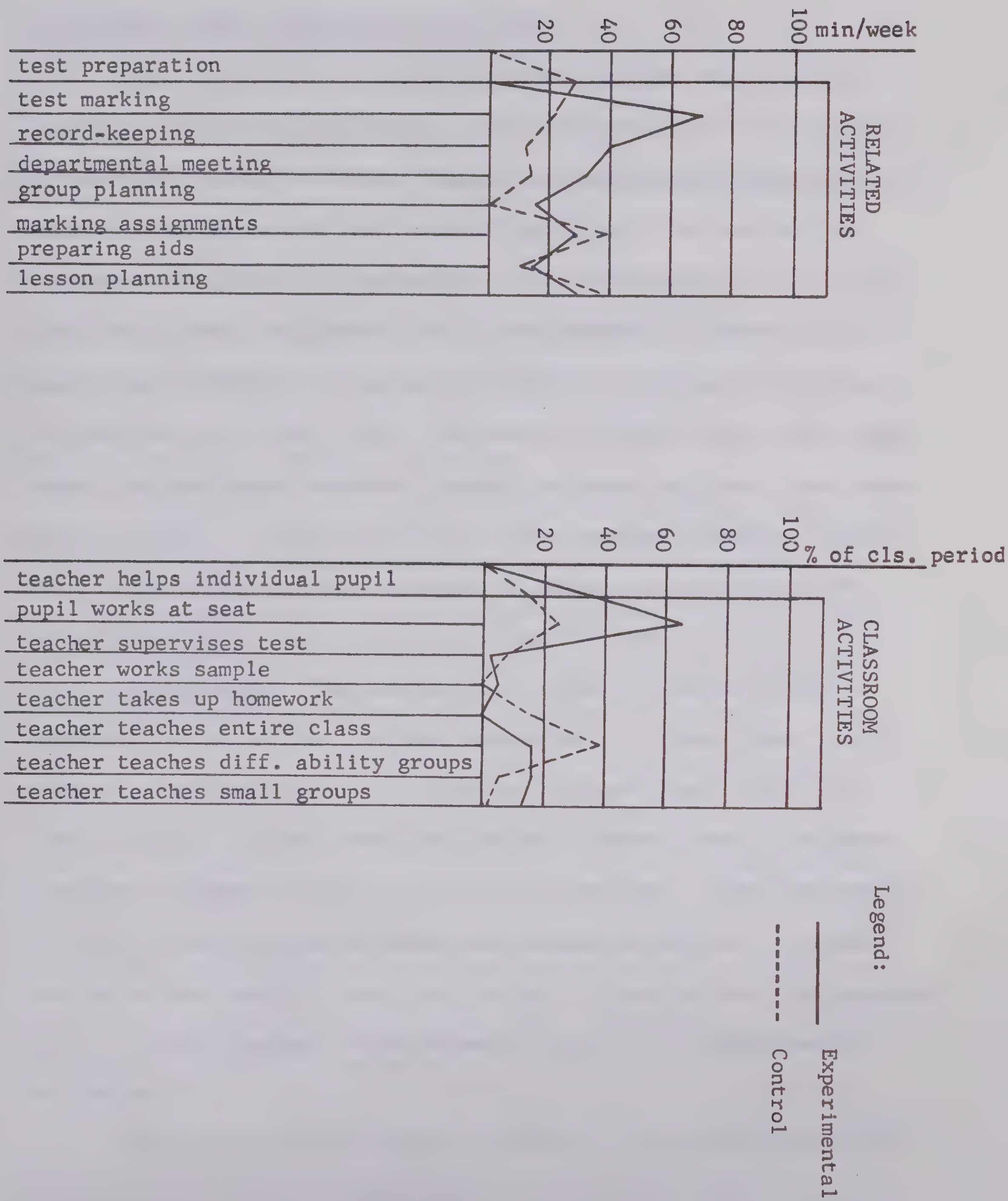


FIGURE XXIX

PROFILE BASED ON LOG SHEET

The Teacher in the Independent Study Class

From the profile it becomes evident that the teacher in this class did not take up any homework, did practically no formal teaching and gave no directed practise. The teacher used formal teacher directed grouping less frequently than informal grouping by the pupils. He addressed the class as an entire group only occasionally, usually to make miscellaneous statements such as announcements. These findings were in part supported by the results from the log sheet in relation to classroom activities. Next, the teacher did very little test supervision. He was never observed cleaning the board and hardly ever seen using the board. He spent much of his time helping individual pupils and a little time with small groups. He could be seen at his desk quite frequently. The teacher moved about the room, but not as much as he normally would. The teacher disciplined the pupils either as individuals or as a class, but not excessively. It would seem disciplining in the independent study setting was much easier than in the control group. This was deceiving for the teachers actually reported they found it more difficult to maintain discipline. Quite frequently he could be seen handing out materials, answering the door or phone, leaving or entering the room. The teacher in this setting also appeared to be a likeable person, for he showed a good deal of affection for his pupils.

Outside of the classroom the teacher in the independent study class was busier than his counterpart in the control group. He spent a considerable amount of time each week in marking tests, keeping records, and planning. The teachers' reactions to the latter few

findings were also accounted for on the questionnaire. They expressed a definite dislike with respect to the excessive time required for marking, record keeping, and planning.

The teacher in the individualized study group worked as a member of a team.

The teachers' reactions to the program were generally favorable. Almost all of them indicated they preferred the individualized study approach.

The Teacher in the Teacher-Taught Class

In this setting the teacher's activities were rather restricted. The teacher in this setting lectured almost the entire period. He also used the question and answer technique during the entire period. His main teaching aid was the blackboard which was used continuously for the duration of the period. Several times during the period the teacher could be seen cleaning the board. For the entire period the whole class was in a formal grouping with the teacher. The teacher's remarks were almost entirely directed to the class as a group. These were basically the only behaviors observed for teachers in the teacher-taught class. The behaviors in this class were quite different from those in the independent study class. The only behavior that remained the same was his display of affection for his pupils.

To compare the teacher in the experimental group to the teacher in the control group can only be done by comparing the teachers in each experimental group to the teachers in the control group. As the profiles indicate the teachers behaved differently in both of the experimental groups than those in the control group. Even the pupil's

behavior was different in many cases. In the independent study class the teacher appeared to have been in the background, while in the teacher-taught class he overwhelmingly was the dominant figure during the entire period. The teachers in the control group fell mainly between the two experimental groups as far as their behaviors were concerned.

The teachers in the experimental group worked and planned as a team, whereas the control group worked and planned as individuals. The workload was considerably heavier for the experimental group than it was for the control group.

Conclusions

The conclusions will be based on the results obtained from the data and, to some extent, the discussions that were held with the teachers during the experiment. The conclusions will be stated in terms of what was required of the teacher, the facilities and will be given under the following categories: the teacher, workload, teacher-aide, facilities, instructional mode, discipline, and team membership.

1. The Teacher

The teachers had to be fully aware of what exactly was expected of them. They had to know the purpose of the independent study and the teacher-taught aspect of the program. The teachers had to know how to use the materials and what the purpose of the materials was. The teachers in the experimental program were very positive in their dealing with the pupils and were able to establish a good working

atmosphere. The teacher in the experimental group acted mainly as a resource person and a diagnostician. He expected the pupils to progress at their own rate. The teachers reported that they were able to get to know their pupils quicker and hence were able to do more for each individual. The noise level was higher in the independent study class than the teachers working in this setting were used to. It hindered them somewhat but they were able to cope with the noise. The teachers of the individualized instruction method had to guard against: the development of poor work habits of the pupils; the loss of the skill of doing neat work; the doing of assignments poorly; and the improper use of instruments.

2. Workload

The teacher in the individualized program had a heavier workload than normally was the case. This load existed in spite of having all the materials prepared for him, of having all the tests prepared, of having pupils help pupils, of having a teacher aide supervising the testing program, and of not having to spend any time on checking homework. Much of the workload was caused by the heavy marking schedule of tests and the extensive record-keeping involved in the experimental program. Planning also required an excessive amount of the teachers' time. In the classroom the teacher was kept busy handing out materials, helping individual pupils and helping small groups. Some of the excesses will disappear once the program has become established, but teachers could never be expected to develop a program of this nature without the assistance of a team of writers and researchers or without adequate release time from teaching duties to do the research and

development. The latter statement was based on the experience the writer had with the development of the materials of the program used in the experiment.

3. Teacher Aide

The teacher aide took quite a burden off the teachers' shoulders. She supervised almost all of the testing and did all of the collating, duplicating, and much of the distributing of the materials prepared by the teachers. The teacher aide did some marking of tests also, but not to any great degree. The teachers had the assistance of one teacher aide in the mathematics department for one half day each day. The teacher aide did not do any of the record keeping, which she could have done if she had had more time.

4. Facilities

It appeared that one large area was required for independent study plus three additional rooms: one for small group instruction, one for testing, and one for the teacher-taught class. The facilities used in the study were not quite adequate which resulted in a classroom being without a teacher most of the time. The latter caused the other teachers to leave their own rooms frequently to check on the room which was without a teacher.

5. Instructional Mode

The teachers had to be able to work mainly on a one-to-one relationship with pupils or in small group situations. Whole class instruction occurred only in the teacher-taught class. The teacher was a diagnostician in that he used the post-tests to determine a

pupil's source of difficulty and to assist a pupil accordingly in his learning experience. The method of individualized instruction was useful in that the teachers were able to do more for the individual pupil and small groups. They reported also that they were able to do more for a class as a whole. The method was also useful in that it permitted the teachers to use a greater variety of teaching methods. The teachers were able to use ability grouping; small group teaching; individual pupil teaching; and the whole class teaching approach. The teachers were able to use the team approach too.

6. Discipline

The nature of the program permitted more pupil interaction and more pupil movement. This required of the teachers a keen sense of control. First, the control of the noise level; second, the control of fraternizing and wasting of time; and third, the control of pupil movement. In each of the cases cited the teacher had to know exactly when to interfere and when not to interfere. Thus the teacher had to have excellent discipline established. From the observations it appeared that it was easier to maintain the discipline in the experimental program, but the teachers reported it to be more difficult to maintain the discipline in the independent study class. From the foregoing it would appear that a teacher should have experience in order to be successful in the experimental program. In a meeting with the teachers it was suggested that a beginning teacher gain experience in a regular classroom first, so that he would know how to handle discipline.

7. Team Membership

The teachers had to be able to work as members of a team. They planned together each unit of instruction as to the teaching strategy and use of materials. The teachers worked as a team when teaching and assisted each other in the supervision of their classes. The teachers mentioned in a meeting that it would have been impossible to see this experiment through if they had not had each other's shoulders to cry on. In this meeting it was also suggested that criticisms, suggestions and disagreements were part of a new approach, but that no matter what the controversy the teachers had to be able to shake hands afterwards. Hence a teacher must be able to be a consistent and dependable member of a team.

The foregoing conclusions have some support from the yet scant information available from other experimental programs using individualized instruction. Among these programs are IPI, Individually Guided Learning and Project PLAN. Also Bloom has given suggestions which agree with the above conclusions. The references made in this paragraph will be handled in more detail in Chapter VI.

CHAPTER VI

INSTRUMENTS, INDIVIDUALIZED INSTRUCTION, TEACHER TRAINING AND FURTHER RESEARCH

Aim of the Study

The purpose of the study was threefold: first, to describe as completely as possible the teacher's role, both in class and out of class, in an individualized instruction setting; second, to develop suitable instruments for the collecting of data necessary for the description of the teacher's role; and third, to make a comparison between the teacher's role in the individualized instruction setting and the teacher's role in regular classroom instruction setting. Due to shortage of resources, the instruments were not rigorously piloted. Therefore any attempt at a reliable statistical analysis on the data gathered with the instruments would be difficult to justify. Yet the writer felt the instruments were adequate to fulfil the aim of the study.

THE INSTRUMENTS DISCUSSED

The discussion in this section is primarily for the benefit of those who might wish to use an instrument similar to the ones used by the writer. The discussion will be in relation to the observation schedule, in the main, and only a brief reference will be made to the log sheet.

The observation schedule was helpful in recalling what occurred

in a classroom. The instrument did have some shortcomings which should be pointed out to anyone who might wish to use it. The first is the lack of any categories referring to the pupils using audio, visual or audio-visual aids. The writer can foresee the case where in an individualized instruction program pupils will be using slides, films, transparencies on an overhead projector, tapes, or even video-tapes. The instrument did have categories in the "formal teaching" section referring to the teacher using the above mentioned audio-visual aides. The categories referring to the use of teaching aids by the pupils should be included in the "laboratory" section of the instrument. The writer feels that future observers are most likely to encounter the use of the above mentioned aids by pupils in a laboratory setting.

The second major shortcoming of the instrument is in section 10 where the affective teacher talk and actions were recorded. The classifications in this section are too broad. The writer has found that the original designer of the OScAR instrument has attempted to refine this aspect of the instrument so that there are now about 52 classifications in this section which he was planning to pilot (Medley, 1966). The writer does not know if the new format proposed by Medley has been finalized as yet.

The third major shortcoming of the instrument is the same as the one reported for the OScAR instrument. None of the categories of the Observation Schedule refer to the content of a statement or the quality of a statement or action, for instance, the teacher asks, "What is the product of 2 X 2?" The instrument simply showed this as: teacher asks question, pupil answers. There is no classification as

to the appropriateness of the question or the quality of the question. Neither did the instrument indicate if the answer given by the pupil was correct or incorrect. In other words the instrument is void of any evaluation of cognitive behaviors.

The above mentioned shortcomings of the Observation Schedule did not interfere with the purpose of the study. The writer did not intend to evaluate; his purpose was to describe what occurred in the classroom using the individualized instruction method. The reasons the writer has confidence in the instrument are two-fold. First, the Observation Schedule shows internal consistency, i.e. when a teacher is lecturing (category 2.1) he will be making problem structuring statements (category 10.3). The two categories should show similar magnitudes in their scores, which indeed occurred. The reader can verify this by referring to the profiles on page 109. Similarly when a teacher is lecturing he will have the class grouped as a whole unit and hence the category referring to this should show the same magnitude also (see category 8.1). Again the writer found this category to be consistent with the previous two (2.1 and 10.3). Second, the Observation Schedule showed consistency in relation to the log sheet on items which were common to both instruments. For instance, the control group reported on the log sheet that they taught the entire class for 40% of the class period and the observer noted that these teachers lectured and used the question and answer technique for 40% of the time.

Finally, the writer wants to refer briefly to the use of the five minute intervals at which observations were made. The notion

of the five minute interval was adopted from the findings of Medley and Mitzel (1958). It is difficult to score an instrument with such a large number of categories in much less time than five minutes. The notion of the technique of scoring used was based on the sampling technique used for statistical purposes. Medley and Mitzel (1958) found that using any observation technique, including the OScAR, would produce an almost perfect record of what actually took place in a classroom whenever twelve or more classroom observations per teacher were made. It was on the basis of the findings by Medley and Mitzel that the writer elected to use the five minute interval. If the length of the interval were extended beyond five minutes the danger of missing too many activities of a short duration would increase considerably. Hence an inaccurate sample would be obtained.

The log sheet not only showed consistency with the Observation Schedule, but it showed internal consistency as well. The latter can be shown by considering the following: the time a classroom teacher has available to deal with small groups or individuals must be equivalent to the time that the remainder of the class is working at their desks. For both the control group and the experimental group the example mentioned works out to within a few minutes. For example, the experimental group reported that the pupils worked at their desks for about 65% of the class time. At the same time they reported that they helped individual pupils for about 35% of the class time; that they taught different ability groups for about 15% of the class time; and that they taught small groups for about 12% of the class time. The last three percentages added up to 62% of the class time, which was

very close to the 65% figure reported for pupils working independently at their desks. It was the two consistencies just mentioned which gave the writer the confidence in the results obtained by the log sheet.

One aspect of the log sheet which will need further research to improve its statistical validity, is the length of the time intervals used. There was no evidence available to the writer which suggested the desirability of the chosen intervals. These were adopted arbitrarily by the writer. When used as a sampling technique it appears that the length of the interval used might be satisfactory provided a large enough sample was taken. For the purpose of this study, the interval used appeared satisfactory because the writer was able to find results similar to the ones obtained on the log sheet with another one of the instruments used in the study, the Observation Schedule.

INDIVIDUALIZED INSTRUCTION SETTING

In the previous chapter the writer attempted to describe as completely as possible what he believed happened in the experimental classroom. On the basis of that description he drew a number of conclusions which were substantiated at that time. In the present section the writer wishes to elaborate on some of the points raised in the previous chapter.

From the conclusions drawn in Chapter V it becomes apparent that a teacher has to acquire certain teaching characteristics in order to be successful in using the individualized instruction method. The teacher plays a role in the individualized classroom different

than the role of the teacher in the regular classroom. The teacher in the individualized classroom is a person who believes firmly in the specific needs of each individual, for he believes that each individual should progress at his own rate in a subject. The teacher using the individualized method must be very positive in his comments and dealings with the pupils. He is an encouraging and reasonably friendly person. The teacher's basic role is one of directing, assisting, advising and diagnosing. The task the teacher faces is one of challenging and encouraging pupils sufficiently so that the pupils will continue to work to their capacity on their own. He has to be capable of dealing with pupils on a one-to-one basis. The nature of the program makes it possible for pupils to work together and to move about freely. Hence the teacher has to have an excellent discipline and a keen sense as to when to curtail pupil movement, pupils working together, and the loudness of the pupils' discussions. Therefore the individualized instruction method places a greater sense of judgment on the shoulders of the teacher, which in turn permits a teacher to be an individual in his own right.

Individualized instruction permits the use of a variety of teaching techniques. The pupil can be taught on a one-to-one basis, in a small group session, in a regular classroom session, and a straight lecture setting. In any one of these approaches a variety of teaching aids can be employed. Hence the teacher must be skilled in a variety of teaching techniques and in the use of a number of teaching aids. The teacher must, therefore, be able to decide which teaching method and which aids to use when, where, and with which pupils. Bloom (1968)

suggested similar ideas to those expressed in the preceeding paragraphs of this section.

The teacher's role in the individualized instruction method is a demanding one in terms of talent and time as well as tact. Facilities would make a good teacher more effective, but are not essential for the success of the program. The genuine interest of the teachers in providing the best instruction possible for each individual pupil is more important than the facilities. The experiment, on which this thesis has been based, was not conducted in an ideal setting as far as facilities were concerned. The adequacy of the facilities fell far short of the ideal. However one external aspect about the experiment which was important to its success besides the crucial role the teacher played was the administrative accommodation of the program. The administration of the school had scheduled the five teachers into the same time slot for grade seven mathematics and had placed the teachers in adjoining rooms. Also the conference room and a storage space were made available for the use of the experimental program. The administration of the school had attempted to facilitate the program in every way possible. Each innovation in teaching brings its own administrative problems and individualized instruction is no exception. To insure optimum results proper administration is essential. The reader who is interested in this aspect would perhaps like to refer to the findings of the study conducted in Wisconsin under Klausmeier, where an attempt is made to develop an administrative design which will facilitate their program of Individually Guided Learning (Klausmeier, 1969).

Another aspect of the teacher's role in individualized instruc-

tion needs emphasizing. From the study it became clear that the teacher had to be able to work as an effective member of a team. The program required a number of teachers doing different aspects of the method simultaneously, for example one teacher would be handling a teacher-taught class, while the others dealt with independent study classes. The division of labor was determined within a team and planning therefore was done on a team basis. The team approach made it possible to use effectively the best talents each teacher possessed. It also provided the necessary support and encouragement as well as a great learning situation for the teachers.

From the experience with the experiment it has become abundantly clear that a teacher carrying a full teaching load can never hope to develop an individualized program of the nature used in this study. The implementation of a new program alone requires more out-of-class time than a regular teaching program does. This showed clearly on the log sheet where the experimental group reported 70 minutes per week per teacher more out-of-class time than the control group did. It appears that two alternatives are available for any group of teachers wishing to go into a program of individualized instruction. Either a team of writers and researchers is assigned to them or sufficient support staff and release time is made available so that the teachers themselves can do the development and research. From the experience the writer had in relation to the preparation of the materials for the experiment it is safe to say that the more time available for development and research the better the materials will be. The role of the teacher remains important however even if a team prepares the

materials. The suggestions from the teachers are vitally important from the standpoint of suitability and proper development of the topics in the materials.

INDIVIDUALIZED INSTRUCTION AND TEACHER TRAINING

To start a new program teachers have to be trained in its use. For existing staff inservice training is normally used and new staff is generally prepared in the teaching method in teacher training institutions. In this section implications for inservice training will be considered first and then implications for teacher training programs will be considered.

The manner in which the experiment came about, from the discussions which were held with the teachers, and from the findings of the study it would appear that the preparation by the teachers of a simplified unit for individualized instruction is perhaps the best form of inservice training to be used. The teachers developed an excellent insight into the philosophy of the program and were capable of handling the more complex approach used for the experiment. From the questionnaire it would appear that whenever the classroom behaviors of a teacher are to be changed it is best to involve the teacher actively in an experimental program using the desired behaviors. All the teachers indicated on the questionnaire that the experimental program affected their regular classroom teaching. They reported that they adapted a number of ideas developed in the experimental program. Yet by far the main requirement is that the teachers are keenly interested and have a vested interest in the program. So the inservice training

program should start with introductory meetings and then those teachers who show a genuine interest should be selected for the preparation of an experimental unit, which will be used by the teachers themselves.

The program of individualized instruction requires a variety of teaching modes and teaching skills. The teacher training institutions basically are concerned with imparting the philosophy of a program, the substance of the subject, and the training of the student teacher in the use of the appropriate teaching skill. Thus any program of teacher training will have to have a section on the philosophy of individualized instruction which perhaps best could be imparted by having pupils develop a simple program. The program of individualized instruction permits each pupil to progress through a course at his own rate and at his own level. A teacher teaching in this setting can no longer prepare himself a week in advance. He has to know exactly every aspect of the course and has to know precisely what each level of ability requires for each topic of the course. It would appear that a teacher using individualized instruction has to be thoroughly familiar with the structure of the course and hence would need a thorough training in the structure of a course, if not more thorough than at present is being given for the regular teaching program. The teacher coming into an individualized program has to be familiar with at least three teaching processes. He has to be able to teach (or tutor) successfully on a one-to-one basis, for much of his teaching effort will be at this level. The present teacher training programs put their major emphasis on large group processes and hence ill prepare a teacher for the one-to-one process. The teacher also has to be skilled in

the small group process and finally he needs skill in large group instruction. The use of laboratory techniques is another aspect of individualized instruction which was not explored in the present study mainly due to lack of equipment and preparation time. The teacher of the future however should be able to use this technique successfully.

The foregoing discussion has concentrated mainly on skills needed by a teacher using a method of individualized instruction. Each of these skills was required in the experiment on which this study was based. The role of the teacher in the individualized instruction program also requires other skills of a teacher. The teacher must be able to diagnose well the needs of each pupil and must be able to direct each pupil's learning activities in such a manner that every pupil achieves to the best of his ability. In other words the student teacher must be trained in diagnosing and prescribing (to strike a parallelism with medicine). Programs of individualized instruction have been developed around this central theme. The IPI program as an example uses this as its main theme of instruction (Scanlon and Bolvin, N.D.). Also Bloom (1968) refers to this as a central part in individualized instruction. Finally the new teacher should be thoroughly trained in group dynamics, because the program of individualized instruction used for this study required teachers to work and plan as a team. In these circumstances a teacher has to know how to act as an efficient member of a team and how to get along in a team situation.

Many of the points raised above are being considered presently in the teacher training institutions. Yet the writer feels that perhaps

a greater emphasis on excellence in these aspects are required. The new teacher has to be skilled in a number of areas, each of which, up to the present time, were considered to be a part of a certain teaching technique and this single teaching technique was being emphasized. It would appear that a teacher needs to be trained more as a generalist as far as different teaching techniques for his subject area are concerned. The reader will find that most of the points mentioned in relation to teacher training also have been mentioned by Southworth (1968) in his model for a teacher training program for individualized instruction.

The method of introducing a new teacher into an existing program of individualized instruction has to be considered carefully. The administrative organization used in the "multiunit" school as developed in Wisconsin (see Klausmeier, 1969) made provision for the introduction of a new teacher into the teaching program. It is a form of internship which might be a good alternative to the one suggested by the teachers involved in the experiment, reported in this thesis. In Wisconsin the new teacher is made a member of a team or "group" of teachers which is under the leadership of a master teacher. The teachers in the experiment suggested that a new teacher be placed in a regular classroom for a couple of years before entering into an individualized teaching program. Yet it is possible that the regular classroom, as it is known now, will no longer exist in the future. Then a form of interning might be necessary. If a program of extended internship came about, the teacher training institutions would possibly be affected in their training programs. It is difficult to foresee at

present what specific changes in their programs would be necessary, but the advent of individualized instruction might very well require a complete change in the role of the teacher training institution. Instead of preparing teachers to work independently as a qualified teacher the preparation might be theoretical only with the working skills being developed in an intern setting.

TOPICS FOR FURTHER RESEARCH

A number of areas have been touched upon in the description of the teacher's role. The introduction of a program of individualized instruction in a given subject area may have many far-reaching effects upon other aspects of a school. It definitely concerns the administration of a school, because the administration has to timetable certain teachers in a specific manner and because of the program certain space allocations may have to be made at the same time. In the experiment five teachers were scheduled to teach the same subject at the same time in adjoining rooms. Many questions related to the just mentioned arrangement remain unanswered. Questions relating to the role of the teacher, the number of teachers needed, and school facilities need to be answered. Further experiments will have to be conducted to determine the conditions which are necessary for individualized instruction to operate efficiently. This research could perhaps be organized under three different headings: the teacher's role, school organization, and school facilities.

In relation to the teacher's role it is necessary to examine which personalities are best suited for individualized instruction

and what variety of personalities make up the best team for individualized instruction. Immediately related to this is the format of the materials which are used for individualized instruction, because each format will place certain restrictions on a teacher's role.

Another broad area of study related to the use of individualized instruction is in relation to school organization. It has to be determined what the optimum number of teachers per team should be in order to make individualized instruction as efficient as possible. It has also to be determined which organization of pupils is best: gradedness or non-gradedness. Another problem which needs research is to determine if teachers should work as subject area groups, i.e. mathematics, or if a team of teachers should handle a group of subjects, i.e. mathematics, science, English, and social studies.

Finally, school facilities are to be considered. It has to be determined which facilities will be the most conducive to the individualized teaching method. It is necessary to determine if a group of adjoining individual classrooms is better than a large open area combined with a number of small adjoining rooms of various sizes or vice versa.

The ideas mentioned for research in the preceding three paragraphs should however not be studied in isolation but should be examined in relation to each other, because each area influences the others and in part determines the others. Hence a large scale study involving a large number of teachers and various school settings would be required. No doubt the writer has overlooked many areas. As for the role of the teacher in individualized instruction the number

of variables to be considered are innumerable. Even if one lists such broad categories as learning styles, teaching styles, subject matter, motivation, the list is lengthy and still incomplete. To relate the variables to the role the teacher would be expected to play boggles the mind. At least the writer appreciates and sympathizes with the feeling expressed by the poet, A. A. Milne:

> Halfway up the stairs
> Isn't up,
> And it isn't down.
> It isn't in the nursery,
> It isn't in the town.
> And all sorts of funny thoughts
> Run round my head:
> It isn't really
> Anywhere!
> It's somewhere else instead!

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APPENDIX A

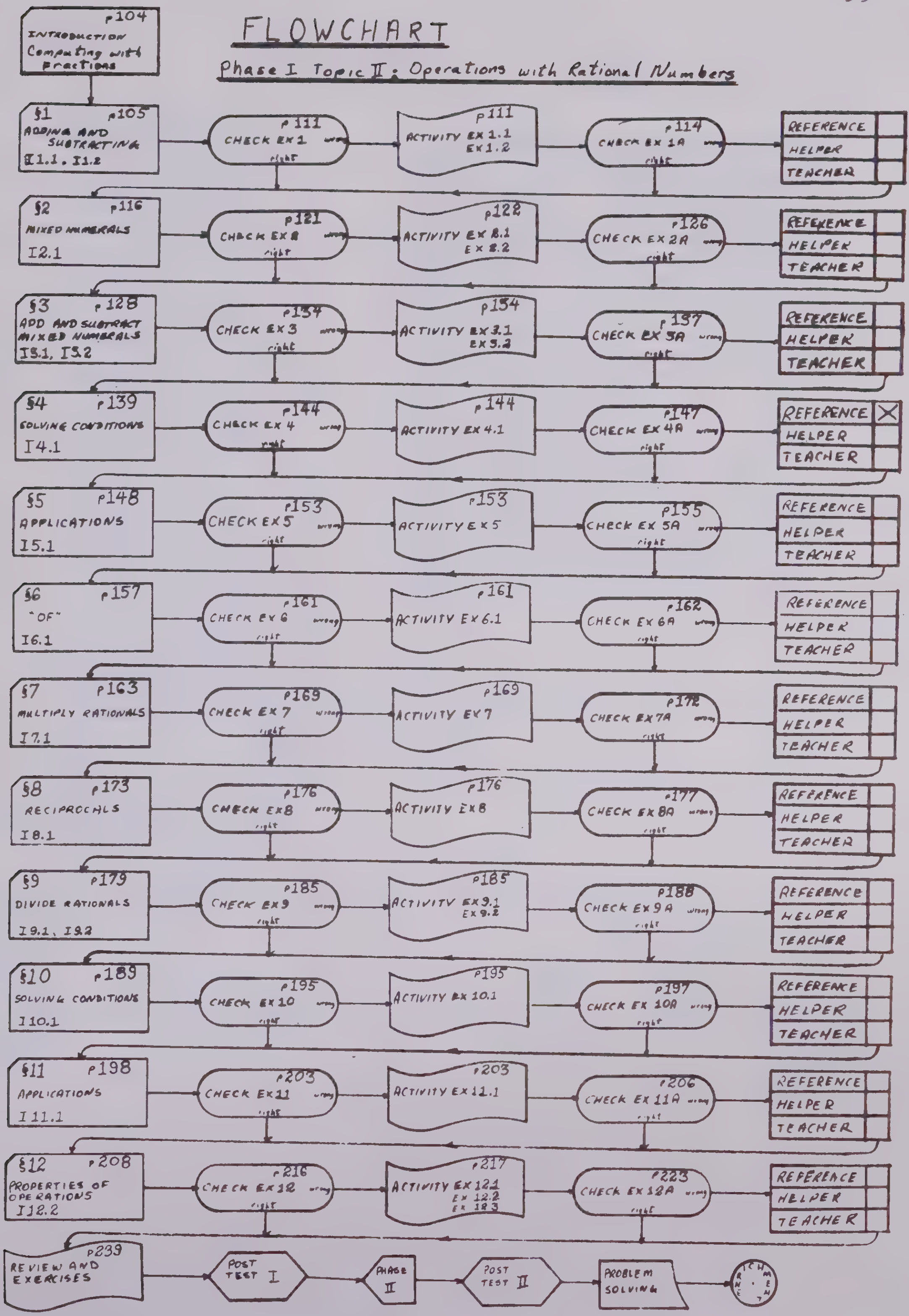
A SAMPLE OF THE MATERIALS USED FOR THE EXPERIMENT

Contents:

- Flow Chart
- Record Page
- Phase I, Topic II, objectives 1-4
- Review Phase I
- Post-Test I
- Phase II, samples for:
  - Basic
  - Intermediate
  - Advanced
- Post-Test II for:
  - Basic
  - Intermediate
  - Advanced
- Challengers

FLOWCHART

Phase I Topic II: Operations with Rational Numbers



TOPIC 2

PHASE I

OPERATIONS WITH RATIONAL NUMBERS

OPERATIONS WITH RATIONAL NUMBERS

INTRODUCTION

In almost every walk of life, at work or at home, people daily have to add, subtract, multiply or divide rational numbers; i.e. they have to use one or more of the four basic operations with rational numbers.

A girl may want to make a recipe for $\frac{2}{3}$ as many people as the directions on the packet indicate or may want to know the cost of $1\frac{3}{4}$ yards of material. A boy may want to know how much he would earn if he increased his paper round by $\frac{1}{3}$ or may want to know the combined length of a piece $1\frac{3}{4}$" long and one $2\frac{5}{8}$" long in a model he is planning.

Operations with rational numbers will also be used often in the work you will be doing in mathematics from now until you finish school. Since you will be using these ideas so much, it is important that you be able to use them accurately and quite quickly.

The operations that you will be studying in this topic are those listed above - addition, subtraction, multiplication and division. You have already met them before.

We will also be concerned with mixed numerals (eg. $4\frac{2}{5}$), with solving conditions involving rational numbers (eg. $n + \frac{2}{3} = \frac{3}{4}$), with reciprocals, (eg. $\frac{2}{3}$ and $\frac{3}{2}$), with properties of rational numbers (eg. commutative property) and with applying rational numbers to answer questions about everyday situations.

OBJECTIVE B1.1

To find the sum of two or more rational numbers named by fractions.

Example

Find the sums and write each as a basic fraction.

A. $\frac{3}{4} + \frac{5}{8} + \frac{1}{4}$

B.
$$\begin{array}{r} \frac{1}{3} \\ \frac{2}{5} \\ + \frac{5}{3} \\ \hline \end{array}$$

Criterion: Method correct in each example and no more than one error in computation.

SOLUTION

A. $\frac{3}{4} + \frac{5}{8} + \frac{1}{4} = \frac{6}{8} + \frac{5}{8} + \frac{2}{8}$

$= \frac{13}{8}$

B.
$$\begin{array}{rcl} \frac{1}{3} & = & \frac{5}{15} \\ \frac{2}{5} & = & \frac{6}{15} \\ + \frac{5}{3} & = & \frac{25}{15} \\ \hline & & \frac{36}{15} = \frac{36 \div 3}{15 \div 3} = \frac{12}{5} \end{array}$$

OBJECTIVE I1.1

To find the sum of two or more rational numbers named by fractions.

Example

Find the sums and write each as a basic fraction.

A. $\frac{1}{5} + \frac{1}{4} + \frac{5}{6}$

B.
$$\begin{array}{r} \frac{1}{9} \\ \frac{7}{6} \\ + \frac{1}{4} \\ \hline \end{array}$$

Criterion: Method correct in each example and no more than one error in computation.

SOLUTION

A. $\frac{1}{5} + \frac{1}{4} + \frac{5}{6} = \frac{12}{60} + \frac{15}{60} + \frac{50}{60}$

$= \frac{77}{60}$

B.
$$\begin{array}{rcl} \frac{1}{9} & = & \frac{4}{36} \\ \frac{7}{6} & = & \frac{42}{36} \\ + \frac{1}{4} & = & \frac{9}{36} \end{array}$$

OBJECTIVE B1.2

To find the difference of two rational numbers named by fractions.

Example

Find the differences and write each as a basic fraction.

A. $\frac{7}{3} - \frac{5}{4}$

B. $\begin{array}{r} \frac{8}{9} \\ -\frac{1}{3} \\ \hline \end{array}$

Criterion: Method correct in each example and no more than one error in computation.

SOLUTION

A. $\frac{7}{3} - \frac{5}{4} = \frac{28}{12} - \frac{15}{12}$
$= \frac{13}{12}$

B. $\begin{array}{rcr} \frac{8}{9} & = & \frac{8}{9} \\ -\frac{1}{3} & = & \frac{3}{9} \\ \hline & & \frac{5}{9} \end{array}$

OBJECTIVE I1.2

To find the difference of two rational numbers named by fractions.

Example

Find the differences and write each as a basic fraction.

A. $\frac{7}{8} - \frac{1}{5}$

B. $\begin{array}{r} \frac{17}{5} \\ -\frac{5}{6} \\ \hline \end{array}$

Criterion: Method correct in each example and no more than one error in computation.

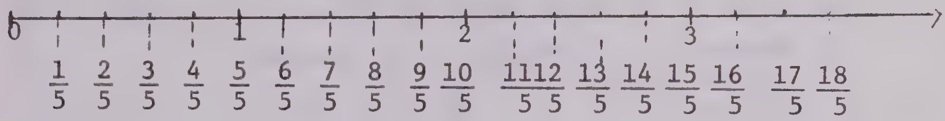
SOLUTION

A. $\frac{7}{8} - \frac{1}{5} = \frac{35}{40} - \frac{8}{40}$
$= \frac{27}{40}$

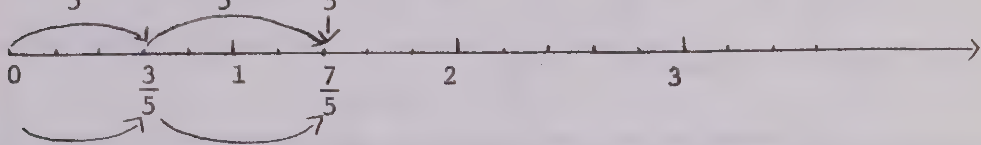
B. $\begin{array}{rcr} \frac{17}{5} & = & \frac{102}{30} \\ -\frac{5}{6} & = & \frac{25}{30} \\ \hline & & \frac{77}{30} \end{array}$

SECTION 1. Operations of addition and subtraction with rational numbers

Suppose we want to add $\frac{3}{5}$ and $\frac{4}{5}$. We can show this on a numberline subdivided into fifths.



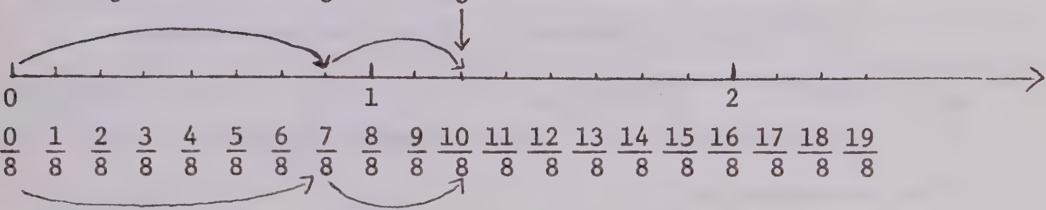
$$\frac{3}{5} \quad + \quad \frac{4}{5} \quad = \quad \frac{7}{5}$$



3 fifths + 4 fifths = 7 fifths

Here is another example, this time with eights. $\frac{7}{8} + \frac{3}{8}$

$$\frac{7}{8} \quad + \quad \frac{3}{8} \quad = \quad \frac{10}{8}$$



7 eights + 3 eights = 10 eights

The sum is $\frac{10}{8}$. Since answers are usually given as basic fractions we reduce $\frac{10}{8}$ to its basic fraction.

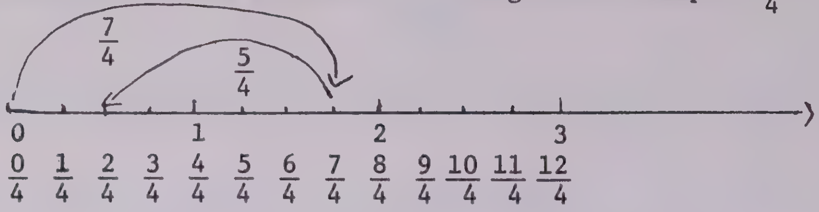
$$\frac{10}{8} = \frac{10 \div 2}{8 \div 2} = \frac{5}{4}$$

1. Use the numberlines above (if necessary) to find

   (1) $\frac{4}{5} + \frac{6}{5}$ (2) $\frac{3}{8} + \frac{9}{8}$

Answers: 1. (1) $\frac{10}{5} = \frac{2}{1} = 2$ (2) $\frac{12}{8} = \frac{3}{2}$

To subtract we do the same sort of thing. For example $\frac{7}{4} - \frac{5}{4}$



7 fourths − 5 fourths = 2 fourths

$$\frac{7}{4} - \frac{5}{4} = \frac{2}{4} = \frac{2 \div 2}{4 \div 4} = \frac{1}{2}$$

2. Write answers to the following in your work book.

(1) $\frac{7}{8} - \frac{3}{8}$ (2) $\frac{11}{16} - \frac{5}{16}$ (3) $\frac{2}{3} + \frac{4}{3}$ (4) $\frac{7}{4} - \frac{7}{4}$

3. Write the answer to $\frac{7}{4} + \frac{1}{2}$

In the examples we have done, the denominators have been the same in both fractions.

What about $\frac{5}{8} + \frac{1}{2}$?

In order to add them the denominators must be the same.

We can use common denominators and the least common denominator (L.C.D.) is usually most convenient. Here, the least common denominator is 8.

$\frac{5}{8} + \frac{1}{2}$ becomes $\frac{5}{8} + \frac{4}{8}$ and this sum is $\frac{9}{8}$.

4. Write the answer to $\frac{2}{3} + \frac{3}{4}$

Here is another example $\frac{2}{5} + \frac{5}{4}$

The L.C.D. is 20

$$\frac{2}{5} = \frac{2 \times 4}{5 \times 4} = \frac{8}{20} \quad \text{and} \quad \frac{5}{4} = \frac{5 \times 5}{4 \times 5} = \frac{25}{20}$$

Thus $\frac{2}{5} + \frac{5}{4} = \frac{8}{20} + \frac{25}{20}$

$= \frac{33}{20}$ (and this is a basic fraction)

Answers: 2: (1) $\frac{4}{8} = \frac{1}{2}$ (2) $\frac{6}{16} = \frac{3}{8}$ (3) $\frac{6}{3} = \frac{2}{1}$ or 2 (4) $\frac{0}{4}$ or 0

3: $\frac{7}{4} + \frac{1}{2} = \frac{7}{4} + \frac{1 \times 2}{2 \times 2} = \frac{7}{4} + \frac{2}{4} = \frac{9}{4}$ 4: $\frac{2}{3} + \frac{3}{4} = \frac{2 \times 4}{3 \times 4} + \frac{3 \times 3}{4 \times 3} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12}$

Yet another example:

$\frac{5}{6} + \frac{9}{10}$ Find the L.C.D. It is 30.

$= \frac{5 \times 5}{6 \times 5} + \frac{9 \times 3}{10 \times 3}$ Find equivalent fractions with this denominator.

$= \frac{25}{30} + \frac{27}{30}$ Add the numerators.

$= \frac{52}{30}$ Is the answer a basic fraction?

$= \frac{52 \div 2}{30 \div 2}$ If not, reduce it to a basic fraction.

$= \frac{26}{15}$

Now a subtraction example. The steps are similar.

$\frac{9}{8} - \frac{5}{6}$ Find the L.C.D. It is 24

$= \frac{9 \times 3}{8 \times 3} - \frac{5 \times 4}{6 \times 4}$ Find equivalent fractions with this denominator. Find the difference of the numerators.

$= \frac{27}{24} - \frac{20}{24}$ The answer is a basic fraction. We are finished.

$= \frac{7}{24}$

Now look at this. Suppose in the last example we had used 48 as the common denominator instead of 24. The steps would be:

$\frac{9}{8} - \frac{5}{6}$

$= \frac{9 \times 6}{8 \times 6} - \frac{5 \times 8}{6 \times 8}$

$= \frac{54}{48} - \frac{40}{48}$

$= \frac{14}{48}$ The result here is not a basic fraction. However, reducing it we get the same as above.

$= \frac{14 \div 2}{48 \div 2}$

$= \frac{7}{24}$

We can use any common denominator when adding or subtracting rational numbers. However, the least common denominator usually gives the result with the least amount of work.

5. Find $\frac{2}{3} + \frac{3}{5} + \frac{7}{4}$

To find the sum of three or more rational numbers, you find the L.C.D. of <u>all</u> the denominators, find equivalent fractions with this denominator and then add the numerators.

For example:

Find

$$\begin{array}{rcccc} \frac{5}{6} & = & \frac{5 \times 6}{6 \times 6} & = & \frac{30}{36} \\ \frac{5}{4} & = & \frac{5 \times 9}{4 \times 9} & = & \frac{45}{36} \\ + \frac{5}{9} & = & \frac{5 \times 4}{9 \times 4} & = & \frac{20}{36} \\ \hline & & & & \frac{95}{36} \end{array}$$

The L.C.D. of 6, 4, and 9 was 36.

Now read OBJECTIVES I1.1 and I1.2 and their examples. These tell you what you are expected to be able to do for this section. When ready turn to and do CHECK EXERCISES 1.1 and 1.2.

Answers: 5: $\frac{2}{3} + \frac{3}{5} + \frac{7}{4} = \frac{2 \times 20}{3 \times 20} + \frac{3 \times 12}{5 \times 12} + \frac{7 \times 15}{4 \times 15} =$

$\frac{40}{60} + \frac{36}{60} + \frac{105}{60} = \frac{181}{60}$

CHECK EXERCISE 1

I1.1 Add the following and write each sum as a basic fraction.

a) $\frac{6}{25} + \frac{3}{10} + \frac{4}{5}$

b) $\frac{5}{8} + \frac{11}{16} + \frac{2}{3}$

c) $\frac{3}{4} + \frac{11}{15} + \frac{7}{12}$

d) $\begin{array}{r} \frac{9}{10} \\ \frac{3}{4} \\ + \frac{2}{5} \\ \hline \end{array}$

e) $\begin{array}{r} \frac{9}{10} \\ \frac{11}{12} \\ + \frac{3}{4} \\ \hline \end{array}$

I1.2 Find the differences and write each as a basic fraction.

a) $\frac{2}{3} - \frac{1}{6}$

b) $\frac{4}{5} - \frac{1}{2}$

c) $\frac{4}{9} - \frac{1}{6}$

d) $\frac{7}{8} - \frac{1}{5}$

e) $\frac{10}{6} - \frac{3}{4}$

- Check your answers with those given at the end of the topic.
- If you are not certain how to add rational numbers, or if you had more than one part incorrect in CHECK EXERCISE 1.1; read section 1 carefully and do activity exercises 1.1.
- If you are not certain how to subtract rational numbers, or if you had more than one part incorrect in CHECK EXERCISE 1.2; read section 1 carefully and do activity exercises 1.2.
- Otherwise, go on to section 2.

Activity Exercises 1.1

1. Using a number line, we can see that $\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$

$\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$

a) Write the fraction that equals $\frac{2}{7} + \frac{4}{7}$.

b) Add the following. Use a number line if you wish, but try to do them without.

i) $\frac{4}{9} + \frac{3}{9}$ ii) $\frac{5}{4} + \frac{2}{4}$ iii) $\frac{3}{10} + \frac{8}{10}$ iv) $\frac{1}{5} + \frac{3}{5}$

$\frac{3}{8} + \frac{1}{4}$ cannot be added as easily because they have different denominators.

But $\frac{1}{4} = \frac{1 \times 2}{4 \times 2} = \frac{2}{8}$. Now we have $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$.

Study the following examples:

a) $\frac{3}{4} + \frac{2}{5}$

$= \frac{3 \times 5}{4 \times 5} + \frac{2 \times 4}{5 \times 4}$ L.C.D. is 20

$= \frac{15}{20} + \frac{9}{20}$

$= \frac{23}{20}$

b) $\frac{1}{4} + \frac{5}{6}$

$= \frac{1 \times 3}{4 \times 3} + \frac{5 \times 2}{6 \times 2}$ L.C.D. is 12

$= \frac{3}{12} + \frac{10}{12}$

$= \frac{13}{12}$

Note: We must have common denominators in order to add rational numbers. The least common denominator is usually most convenient.

c) Add the following. Be sure you have a common denominator.

i) $\frac{1}{2} + \frac{3}{4}$

ii) $\frac{2}{3} + \frac{1}{6}$

iii) $\frac{8}{5} + \frac{4}{3}$

iv) $\frac{4}{7} + \frac{5}{9}$

v) $\frac{11}{10} + \frac{11}{7}$

Adding $\frac{3}{10} + \frac{3}{10}$ we get $\frac{6}{10}$. But $\frac{6}{10}$ can be reduced to a basic fraction of $\frac{3}{5}$. ($\frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$).

Note: We always reduce a fraction to its basic fraction before considering the question finished.

d) Add the following. Be sure they are reduced to basic fractions when necessary.

i) $\frac{5}{12} + \frac{1}{4}$

ii) $\frac{1}{15} + \frac{5}{6}$

iii) $\frac{3}{10} + \frac{7}{8}$

Adding three or more rational numbers is done just as for adding two rational numbers. Be sure they ALL are expressed with common denominators and the final answers are BASIC FRACTIONS.

$\frac{2}{3} + \frac{4}{5} + \frac{5}{6}$

$= \frac{20}{30} + \frac{24}{30} + \frac{25}{30}$ - common denominators (L.C.M. is 30)

$= \frac{69}{30}$

$= \frac{23}{10}$ - basic fraction

We can also use column form when adding 2 or more fractions.

$$\begin{array}{rcrl} \frac{3}{8} & = & \frac{9}{24} & \\ \frac{5}{6} & & \frac{20}{24} & \text{L.C.D. is 24} \\ + \ \frac{2}{3} & & + \ \frac{16}{24} & \\ \hline & & \frac{45}{24} & \\ & & = \frac{15}{8} & \text{basic fraction} \end{array}$$

e) Add the following and express the sums as basic fractions.

i) $\frac{5}{6} + \frac{1}{2} + \frac{2}{3}$

ii) $\frac{1}{2} + \frac{3}{8} + \frac{1}{4}$

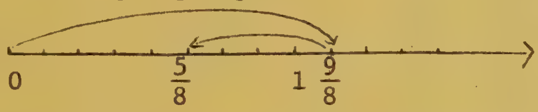
iii) $\frac{3}{8} + \frac{5}{12} + \frac{8}{15}$

iv) $\begin{array}{r} \frac{2}{3} \\ \frac{2}{5} \\ + \ \frac{7}{15} \\ \hline \end{array}$

v) $\begin{array}{r} \frac{2}{3} \\ \frac{3}{4} \\ + \ \frac{5}{6} \\ \hline \end{array}$

Activity Exercises 1.2

We can also use the number line to find the difference between two rational numbers. $\frac{9}{8} - \frac{4}{8} = \frac{5}{8}$



a) Subtract the following. Use a number line if necessary, but try to do it without.

i) $\frac{5}{9} - \frac{4}{9}$ ii) $\frac{12}{5} - \frac{8}{5}$ iii) $\frac{7}{8} - \frac{5}{8}$

In order to subtract $\frac{4}{5}$ from $\frac{9}{10}$ we must express both fractions with a common denominator.

$$\frac{9}{10} - \frac{4}{5}$$

$$= \frac{9}{10} - \frac{8}{10} \qquad \left(\frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10} \right)$$

$$= \frac{1}{10}$$

b) Subtract the following. Use the least common denominator.

i) $\frac{9}{5} - \frac{2}{15}$ ii) $\frac{5}{8} - \frac{2}{6}$ iii) $\frac{5}{7} - \frac{2}{5}$

In the following example we find we get an answer of $\frac{2}{12}$.

$$\frac{7}{12} - \frac{5}{12} = \frac{2}{12}$$

But, we do not leave the answer as $\frac{2}{12}$. We reduce it to its BASIC FRACTION.

$$\frac{2}{12} = \frac{2 \div 2}{12 \div 2} = \frac{1}{6}$$

Thus $\frac{7}{12} - \frac{5}{12} = \frac{2}{12}$

$$= \frac{1}{6}$$

We can also subtract in column form.

$$\begin{array}{rcr} \frac{4}{5} & = & \frac{4}{30} \\ -\ \frac{1}{10} & & -\ \frac{3}{30} \\ \hline & & \frac{1}{30} \end{array}$$

NOTE: ALWAYS EXPRESS THE ANSWER AS A BASIC FRACTION.

c) Find the differences and express each as a basic fraction.

i) $\frac{7}{16} - \frac{1}{4}$

ii) $\frac{3}{10} - \frac{1}{20}$

iii) $\frac{9}{20} - \frac{1}{5}$

iv) $\begin{array}{r} \frac{3}{4} \\ -\ \frac{2}{3} \\ \hline \end{array}$

v) $\begin{array}{r} \frac{9}{5} \\ -\ \frac{4}{7} \\ \hline \end{array}$

- Check your answers with those given at the end of the topic.
- Read objective I1.1 and I2.1.
- If you feel you are ready, do CHECK EXERCISE 1A.

CHECK EXERCISE 1A

I1.1 Add the following and write each sum as a basic fraction.

a) $\frac{1}{4} + \frac{2}{9} + \frac{5}{12}$

b) $\frac{1}{4} + \frac{4}{25} + \frac{3}{10}$

c) $\frac{3}{14} + \frac{4}{21} + \frac{5}{28}$

d) $\begin{array}{r} \frac{1}{6} \\ \frac{2}{3} \\ +\ \frac{0}{5} \\ \hline \end{array}$

e) $\begin{array}{r} \frac{3}{8} \\ \frac{1}{6} \\ +\ \frac{5}{12} \\ \hline \end{array}$

I1.2 Find the differences and write each as a basic fraction.

a) $\frac{7}{12} - \frac{1}{4}$

b) $\frac{7}{15} - \frac{1}{6}$

c) $\frac{5}{8} - \frac{1}{3}$

d) $\frac{11}{16} - \frac{7}{12}$

e) $\frac{5}{6} - \frac{8}{15}$

- Check your answers with those given at the end of the topic.
- If you are not sure how to add or subtract rational numbers, or if you had more than one error in each CHECK EXERCISE, check <u>Modern School Mathematics</u> pp. 336-340 or ask a student helper or ask your teacher.
- Otherwise, go on to section 2.

OBJECTIVE B2.1

(1) To write a fraction with numerator greater than denominator as a mixed numeral.

(2) To write a mixed numeral as a fraction.

Example

(1) Write as a mixed numeral $\frac{35}{8}$

(2) Write as a fraction $7\frac{2}{3}$

SOLUTION

$$\frac{35}{8} = \frac{32}{8} + \frac{3}{8} = 4\frac{3}{8}$$

$$7\frac{2}{3} = \frac{21}{3} + \frac{2}{3} = \frac{23}{3}$$

Criterion: Correct mixed numeral and fraction.

OBJECTIVE I2.2

a) To use fractions to justify that a given fraction has a particular mixed numeral.

b) To use fractions to justify that a given mixed numeral has a particular fraction.

Example

a) Use fractions to justify that the mixed numeral for $\frac{22}{5}$ *is* $4\frac{2}{5}$.

b) Use fractions to justify that the fraction for $5\frac{2}{3}$ *is* $\frac{17}{3}$.

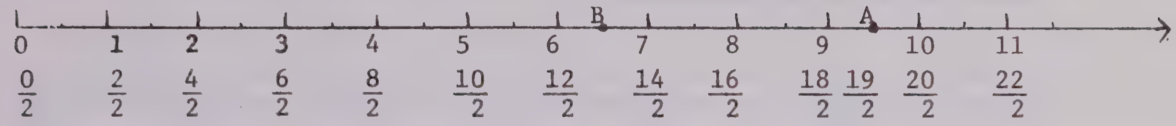
Criterion: Correct justifications.

SOLUTION

a) $$\frac{22}{5} = \frac{20}{5} + \frac{2}{5} = \frac{20 \div 5}{5 \div 5} + \frac{2}{5} = \frac{4}{1} + \frac{2}{5} = 4 + \frac{2}{5} = 4\frac{2}{5}$$

b) $$5\frac{2}{3} = 5 + \frac{2}{3} = \frac{5}{1} + \frac{2}{3} = \frac{5 \times 3}{1 \times 3} + \frac{2}{3} = \frac{15}{3} + \frac{2}{3} = \frac{17}{3}$$

Section 2. Mixed numerals and fractions



1. Write two ways of naming the rational number associated with point B on the number line.

The rational number associated with point A on the number line above may be named in two ways; $\frac{19}{2}$ or $9\frac{1}{2}$. The second of the names is the one often used in everyday situations; e.g. $9\frac{1}{2}$ inches, $9\frac{1}{2}$ years, etc.

On the number line subdivided into halves, $9\frac{1}{2}$ means 9 whole divisions plus one of the halves subdivisions. This is the same as 19 subdivisions, each one half.

$9\frac{1}{2}$ is called a mixed numeral because it contains both a whole number numeral and a fraction.

$9\frac{1}{2}$ fraction — whole number numeral

$9\frac{1}{2}$ mixed numeral

$9\frac{1}{2} = 9 + \frac{1}{2}$ meaning

When we get to applications of rational numbers to answer questions about everyday situations we will find that mixed numerals are often used. We will also need to be able to convert between mixed numerals and fractions so let's see how this is done.

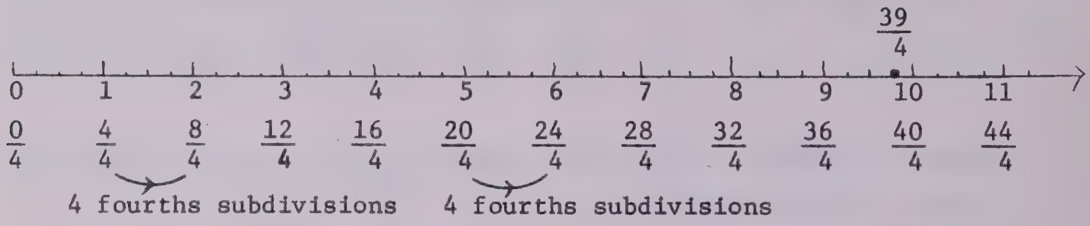
Fractions to mixed numerals

Of course, it is only when the numerator of the fraction is greater than the denominator that this can be done.

2. Write the mixed numeral for $\frac{17}{4}$.

Answers: 1: $\frac{13}{2}$ or $6\frac{1}{2}$ 2: 16 fourths are 4. Thus $\frac{17}{4} = 4\frac{1}{4}$.

Consider the fraction $\frac{39}{4}$. In terms of a number line subdivided into fourths, the point corresponding to $\frac{39}{4}$ is 39 fourths subdivisions from the point corresponding to 0.



It takes 4 fourths subdivisions to make each whole division. How many whole subdivisions are included in 39 fourths subdivisions? We find out by dividing 39 by 4.

$$\begin{array}{r} 9 \\ 4\overline{)39} \\ \underline{36} \\ 3 \end{array}$$

39 fourths subdivisions is 9 whole divisions and 3 fourths subdivisions.

i.e. $\frac{39}{4} = 9\frac{3}{4}$

This can also be justified, using fractions, as follows:

$$\frac{39}{4} = \frac{36}{4} + \frac{3}{4} = \frac{36 \div 4}{4 \div 4} + \frac{3}{4} = \frac{9}{1} + \frac{3}{4} = 9 + \frac{3}{4} = 9\frac{3}{4}$$

Let's try $\frac{104}{16}$ and obtain the mixed numeral for it.

First divide, to find the number of wholes.

$$\begin{array}{r} 6 \\ 16\overline{)104} \\ \underline{96} \\ 8 \end{array} \quad 6\frac{8}{16}$$

But $\frac{8}{16}$ can be reduced to the basic fraction $\frac{1}{2}$

Thus $\frac{104}{16} = 6\frac{8}{16} = 6\frac{1}{2}$

Justification using fractions

$$\begin{aligned} \frac{104}{16} &= \frac{96}{16} + \frac{8}{16} \\ &= \frac{96 \div 16}{16 \div 16} + \frac{8 \div 8}{16 \div 8} \\ &= \frac{6}{1} + \frac{1}{2} \\ &= 6 + \frac{1}{2} \\ &= 6\frac{1}{2} \end{aligned}$$

3. Write $\frac{100}{8}$ as a mixed numeral.

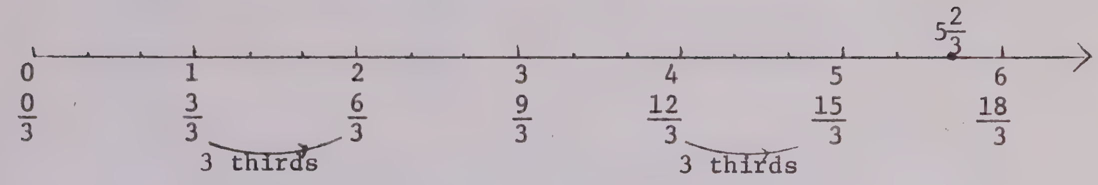
<u>Mixed numerals to fractions</u>

This can be done for every mixed numeral.

Answers: 3: $\begin{array}{r} 12 \\ 8\overline{)100} \end{array}$ remainder 4 $\quad \frac{100}{8} = 12\frac{4}{8} = 12\frac{1}{2}$

4. Write the fraction for $4\frac{2}{3}$.

Consider the mixed numeral $5\frac{2}{3}$.



Each whole division has 3 thirds subdivisions.

5 whole divisions have 5 x 3 = 15 thirds subdivisions.

$5\frac{2}{3}$ has 15 thirds subdivisions plus 2 more thirds subdivisions;

i.e. 17 thirds subdivisions.

Thus $5\frac{2}{3} = \frac{17}{3}$

This can also be justified using fractions as follows:

$$5\frac{2}{3} = 5 + \frac{2}{3} = \frac{5}{1} + \frac{2}{3} = \frac{5 \times 3}{3 \times 3} + \frac{2}{3} = \frac{15}{3} + \frac{2}{3} = \frac{17}{3}$$

Let's try $4\frac{7}{12}$ and obtain the fraction for it.

First multiply to find the number of twelfths in 4

4 is 4 x 12 = 48 twelfths

$4\frac{7}{12}$ is (48 + 7) twelfths

$4\frac{7}{12} = \frac{55}{12}$

Justification using fractions

$$\begin{aligned} 4\frac{7}{12} &= 4 + \frac{7}{12} \\ &= \frac{4}{1} + \frac{7}{12} \\ &= \frac{4 \times 12}{1 \times 12} + \frac{7}{12} \\ &= \frac{48}{12} + \frac{7}{12} \\ &= \frac{55}{12} \end{aligned}$$

5. Write the mixed numeral for $9\frac{8}{9}$.

For a number to be correctly named by a mixed numeral, the fraction part must have numerator less than denominator.

$3\frac{5}{4}$ is not written correctly. It should be $4\frac{1}{4}$.

Answers: 4: 4 is 12 thirds. Thus $4\frac{2}{3}$ is $\frac{14}{3}$.

5: 9 is 9 x 9 = 81 ninths. $9\frac{8}{9} = \frac{89}{9}$

Now read OBJECTIVES B2.1 and I2.2 and their examples. These tell you what you are expected to be able to do for this section. When ready, turn to and do CHECK EXERCISES 2.1 and 2.2.

CHECK EXERCISE 2

B2.1 i) Write the following fractions as mixed numerals:

a) $\frac{37}{5}$ d) $\frac{13}{7}$

b) $\frac{9}{4}$ e) $\frac{16}{9}$

c) $\frac{11}{3}$

ii) Write the following mixed numerals as fractions:

a) $1\frac{3}{4}$ d) $4\frac{4}{9}$

b) $2\frac{5}{7}$ e) $5\frac{3}{8}$

c) $1\frac{8}{11}$

I2.2 i) Use fractions to justify that the mixed numeral for:

a) $\frac{15}{4}$ is $3\frac{3}{4}$

b) $\frac{22}{7}$ is $3\frac{1}{7}$

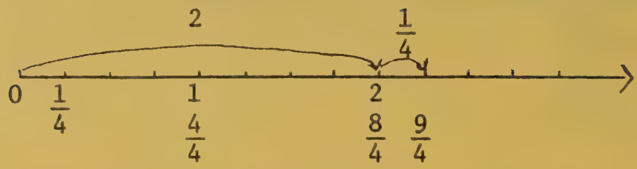
ii) Use fractions to justify that the fraction for:

a) $3\frac{3}{5}$ is $\frac{18}{5}$

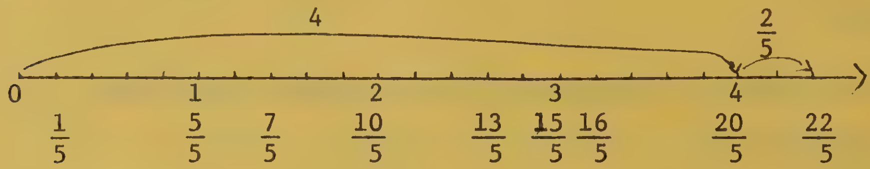
b) $2\frac{1}{4}$ is $\frac{9}{4}$

- Check your answers with those given at the end of the topic.
  You were successful on CHECK EXERCISE B2.1 if you had at least 4 of the parts correct in both 2.1 (i) and 2.1 (ii).
  You were successful on CHECK EXERCISE I2.2 if you had 3 of the 4 parts correct and all steps were shown.
- If you are not sure how to do either of the above CHECK EXERCISES or if you were unsuccessful on CHECK EXERCISE B2.1 read section 2 and do activity exercise 2.1.
  If you were unsuccessful on CHECK EXERCISE I2.2 read section 2 and do activity exercise 2.2.
- Otherwise, go on to section 3.

Activity Exercise 2.1



From the number line, we can see that $\frac{9}{4}$ means 2 whole units on the number line plus one-quarter of the next unit. This could be written as $2\frac{1}{4}$, which means $2 + \frac{1}{4}$. $2\frac{1}{4}$ is said to be a <u>mixed numeral</u> as it contains both a whole number and a fraction.



On the number line above we see that $\frac{22}{5}$ has four whole units divided into fifths plus $\frac{2}{5}$ of the next unit. ie. $\frac{22}{5} = 4\frac{2}{5}$

1. Using the above number line, write mixed numerals for:

   a) $\frac{7}{5}$ b) $\frac{13}{5}$ c) $\frac{16}{5}$

   Of course we do not want to always draw a number line to determine the mixed numeral. We can use the following method which involves division.

   We can find the mixed numeral for $\frac{22}{5}$ by dividing 22 by 5.

$$\begin{array}{r} 4 \\ 5\,\overline{)\,22} \\ \underline{20} \\ 2 \end{array} \rightarrow 4\frac{2}{5}$$

   Four whole units and $\frac{2}{5}$ of the next unit.

   Similarly $\frac{26}{5} = 5\frac{1}{5}$ because

$$\begin{array}{r} 5 \\ 5\,\overline{)\,26} \\ \underline{25} \\ 1 \end{array} \quad \text{i.e. } 5\frac{1}{5}$$

2. Using the division method write mixed numerals for:

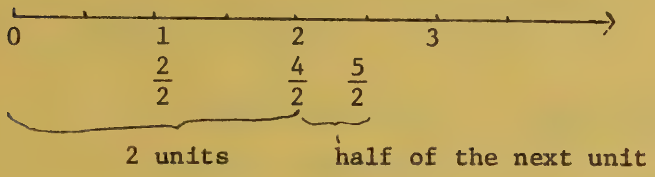
a) $\frac{15}{4}$ d) $\frac{31}{6}$

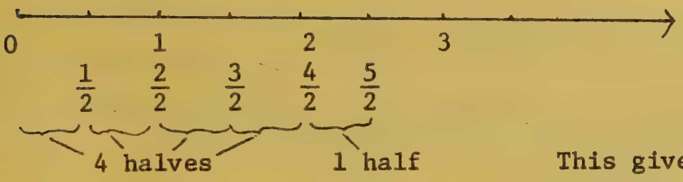
b) $\frac{17}{3}$ e) $\frac{24}{7}$

c) $\frac{21}{8}$

Given the mixed numeral we also want to be able to express it as a fraction.

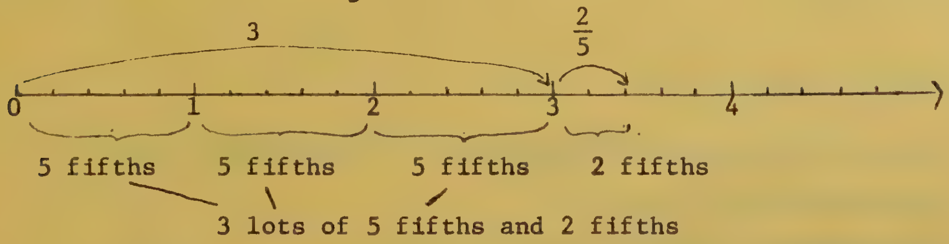
$2\frac{1}{2}$ means $2 + \frac{1}{2}$ or 2 units plus $\frac{1}{2}$ of the next unit.





This gives 5 halves or $\frac{5}{2}$.

Study the following example in which we write a fraction for the mixed numeral $3\frac{2}{5}$.



i.e. (3 x 5) fifths + 2 fifths

i.e. $\frac{15}{5} + \frac{2}{5}$

i.e. $\frac{17}{5}$

3. Write a fraction for $2\frac{3}{4}$.

Again we want to be able to write fractions equivalent to mixed numerals without having to draw a number line. Looking at the mixed numeral $3\frac{2}{5}$ we see that it is made up of 3 and $\frac{2}{5}$. To find the number of fifths in 3 we multiply 3 by 5. This gives us 15 fifths in the 3 plus 2 more fifths in the $\frac{2}{5}$. Thus we have 15 + 2 fifths, i.e. 17 fifths or $\frac{17}{5}$.

To do this quivkly we can think $3\frac{2}{5}$ = [(5 x 3) + 2] fifths

= [15 + 2] fifths

= $\frac{17}{5}$

$4\frac{2}{3}$ = [(3 x 4) + 2] thirds

= 12 + 2 thirds

= $\frac{14}{3}$

4. Write fractions for the following mixed numerals:

a) $5\frac{2}{5}$

b) $3\frac{2}{3}$

c) $1\frac{4}{7}$

d) $5\frac{1}{4}$

e) $2\frac{7}{9}$

Activity Exercise 2.2

We can use <u>fractions</u> to <u>justify</u> that a given fraction has a particular mixed numeral.

For example:

$$\frac{15}{4} = \frac{12}{4} + \frac{3}{4}$$
$$= \frac{12 \div 4}{4 \div 4} + \frac{3}{4}$$
$$= \frac{3}{1} + \frac{3}{4}$$
$$= 3 + \frac{3}{4}$$
$$= 3\frac{3}{4}$$

all these steps are necessary for the justification

1. Write out and complete the next example.

$$\begin{aligned}\frac{23}{5} &= \frac{\square}{5} + \frac{\square}{5}\\ &= \frac{20 \div \square}{5 \div \square} + \frac{3}{5}\\ &= \frac{\square}{\square} + \frac{3}{5}\\ &= \square + \square\\ &= \square\end{aligned}$$

(note: we want to form two fractions such that one may be simplified to a whole number <u>and</u> the other has numerator less than denominator).

2. Justify that each of the following fractions is the given mixed numeral:

a) $\frac{37}{6}$ is the mixed numeral $6\frac{1}{6}$.

b) $\frac{17}{3}$ is the mixed numeral $5\frac{2}{3}$.

We can also use <u>fractions</u> to <u>justify</u> that a given mixed numeral has a particular fraction.

For example:

$$\begin{aligned}4\tfrac{2}{3} &= 4 + \frac{2}{3}\\ &= \frac{4}{1} + \frac{2}{3}\\ &= \frac{4 \times 3}{1 \times 3} + \frac{2}{3}\\ &= \frac{12}{3} + \frac{2}{3}\\ &= \frac{14}{3}\end{aligned}$$

all these steps are needed for the justification

3. Write out and complete the following example.

$$\begin{aligned}3\tfrac{5}{7} &= \square + \square\\ &= \frac{\square}{\square} + \frac{5}{7}\\ &= \frac{3 \times \square}{1 \times \square} + \frac{5}{7}\\ &= \frac{\square}{\square} + \frac{5}{7}\\ &= \frac{\square}{\square}\end{aligned}$$

(note: we want to write the whole number as a fraction with the same denominator as the fraction in the mixed numeral).

4. Justify that the following mixed numerals have the given fractions:

a) $2\frac{3}{4}$ has the fraction $\frac{11}{4}$

b) $6\frac{5}{8}$ has the fraction $\frac{53}{8}$

- Check your answers with those given at the end of the topic.
- Read objective B2.1 and I2.2.
- Do CHECK EXERCISE 2A.

CHECK EXERCISE 2A

B2.1 i) Write the following fractions as mixed numerals.

a) $\frac{34}{15}$ d) $\frac{10}{7}$

b) $\frac{48}{17}$ e) $\frac{17}{5}$

c) $\frac{27}{10}$

ii) Write the following mixed numerals as fractions.

a) $3\frac{5}{8}$ d) $1\frac{11}{12}$

b) $7\frac{5}{6}$ e) $5\frac{15}{22}$

c) $3\frac{7}{15}$

I2.2 i) Use fractions to justify that the mixed numeral for

a) $2\frac{8}{3}$ is $9\frac{1}{3}$

b) $\frac{20}{9}$ is $2\frac{2}{9}$

ii) Use fractions to justify that the fraction for

a) $8\frac{5}{6}$ is $\frac{53}{6}$

b) $7\frac{3}{4}$ is $\frac{31}{4}$

- Check your answers with those given at the end of the topic.
You were successful on CHECK EXERCISE B2.1 if you had at least 4 parts correct in each of 2.1(i) and 2.1(ii).
You were successful on CHECK EXERCISE I2.2 if you had 3 of the 4 parts correct and indicated all steps.

- If you are not sure how to work with mixed numerals or if you were not successful; check reference book - <u>Modern School Mathematics</u> pp. 345, 346 or ask a student helper or ask your teacher.

- Otherwise, go on to section 3.

OBJECTIVE B3.1

To find the sum of two or more rational numbers named by fractions or mixed numerals.

Example

Find the sums and write each as a mixed numeral with fraction part as a basic fraction.

A. $\begin{array}{r} 4\frac{1}{8} \\ +\ 2\frac{1}{4} \\ \hline \end{array}$

B. $5\frac{1}{2} + 2\frac{2}{3} + \frac{3}{4}$

Criterion: Method correct in each example and no more than one error in computation.

SOLUTION

A.
$$\begin{array}{rcr} 4\frac{1}{8} & = & 4 + \frac{1}{8} \\ 2\frac{1}{4} & = & 2 + \frac{2}{8} \\ \hline & & 6\frac{3}{8} \end{array}$$

B.
$$\begin{aligned} 5\frac{1}{2} + 2\frac{2}{3} + \frac{3}{4} &= 5\frac{6}{12} + 2\frac{8}{12} + \frac{9}{12} \\ &= 7 + \frac{23}{12} \\ &= 7 + 1\frac{11}{12} \\ &= 8\frac{11}{12} \end{aligned}$$

OBJECTIVE I3.1

To find the sum of two or more rational numbers named by fractions or mixed numerals.

Example

Find the sums and write each as a mixed numeral with fraction part as a basic fraction.

A. $\begin{array}{r} 1\frac{1}{4} \\ \frac{2}{5} \\ +\ 1\frac{2}{3} \\ \hline \end{array}$

B. $\frac{4}{5} + 3 + 4\frac{1}{3} + \frac{7}{10}$

Criterion: Method correct in each example and no more than one error in computation.

SOLUTION

A. $1\frac{1}{4} = 1 + \frac{15}{60}$ $\frac{2}{5} = \frac{24}{60}$ $1\frac{2}{3} = 1 + \frac{40}{60}$ $= 1 + \frac{79}{60} = 1 + 1 + \frac{19}{60} = 2\frac{19}{60}$	B. $\frac{4}{5} + 3 + 4\frac{1}{3} + \frac{7}{10} = \frac{24}{30} + 3 + 4\frac{10}{30} + \frac{21}{30}$ $= 7 + \frac{55}{30}$ $= 7 + 1\frac{25}{30}$ $= 8\frac{25}{30}$ $= 8\frac{5}{6}$

OBJECTIVE B3.2

To find the difference of two rational numbers named by fractions or mixed numerals.

Example

Find the differences and write the fraction parts as basic fractions.

A. $3\frac{2}{3} - 1\frac{2}{5}$ B. $4\frac{1}{4}$ $- 3\frac{5}{6}$

Criterion: Method correct in each example and no more than one error in computation.

SOLUTION

A. $3\frac{2}{3} - 1\frac{2}{5} = 3\frac{10}{15} - 1\frac{6}{15}$ $= 2\frac{4}{15}$	B. $4\frac{1}{4} = 4 + \frac{3}{12} = 3 + \frac{15}{12}$ $- 3\frac{5}{6} = 3 + \frac{10}{12} = 3 + \frac{10}{12}$ $\frac{5}{12}$

OBJECTIVE I3.2

To find the difference of two rational numbers named by fractions or mixed numerals.

Example

Find the differences and write the fraction parts as basic fractions:

A. $3\frac{5}{8} - 2\frac{4}{5}$ *B.* $6\frac{5}{9}$ $- \frac{3}{4}$

Criterion: *Method correct in each example and no more than one error in computation.*

SOLUTION

A. $3\frac{5}{8} - 2\frac{4}{5} = 3\frac{25}{40} - 2\frac{32}{40}$

$= 1 + \frac{25}{40} - \frac{32}{40}$

$= \frac{65}{40} - \frac{32}{40}$

$= \frac{33}{40}$

B.

$$\begin{array}{rcrcr} 6\frac{5}{9} & = & 6 + \frac{20}{36} & = & 5 + \frac{56}{36} \\ -\underline{\frac{3}{4}} & = & \underline{\frac{27}{36}} & = & \underline{\frac{27}{36}} \\ & & & & 5\frac{29}{36} \end{array}$$

Section 3. Addition and Subtraction involving rational numbers named by mixed numerals

Example 1   Read through this example and the comments

$$\begin{aligned} & 3\tfrac{1}{4} + 2\tfrac{2}{3} \\ = \ & 3 + \tfrac{1}{4} + 2 + \tfrac{2}{3} \end{aligned}$$

Mixed numerals represent the sum of a number named by a whole number numeral and a number named by a fraction.

$$\begin{aligned} = \ & 3 + 2 + \tfrac{1}{4} + \tfrac{2}{3} \\ = \ & 5 + \tfrac{3}{12} + \tfrac{8}{12} \end{aligned}$$

Add the whole numbers and add the numbers named by the fractions. The L.C.D. is 12.

$$\begin{aligned} = \ & 5 + \tfrac{11}{12} \\ = \ & 5\tfrac{11}{12} \end{aligned}$$

Write the result as a mixed numeral.

You can probably do an example like this without writing down steps 2, 3 or 5

$$\begin{aligned} & 3\tfrac{1}{4} + 2\tfrac{2}{3} \\ = \ & 5 + \tfrac{3}{12} + \tfrac{8}{12} \\ = \ & 5\tfrac{11}{12} \end{aligned}$$

Example 2   Read through this example and the comments

$$\begin{aligned} & 3\tfrac{3}{4} + 2\tfrac{2}{3} \\ = \ & 5 + \tfrac{9}{12} + \tfrac{8}{12} \end{aligned}$$

Add the whole numbers and add the numbers named by the fractions. The L.C.D. is 12.

$$= \ 5 + \tfrac{17}{12}$$

$\frac{17}{12}$ is $1\,\frac{5}{12}$ as a mixed numeral, i.e. $1 + \frac{5}{12}$

$$\begin{aligned} = \ & 5 + 1 + \tfrac{5}{12} \\ = \ & 6\,\tfrac{5}{12} \end{aligned}$$

Add the whole numbers.

1. Find the sum: $5\frac{3}{4} + \frac{5}{6}$

Answer: 1:

$$\begin{aligned} & 5\tfrac{3}{4} + \tfrac{5}{6} \\ = \ & 5 + \tfrac{9}{12} + \tfrac{10}{12} \\ = \ & 5 + \tfrac{19}{12} \\ = \ & 5 + 1 + \tfrac{7}{12} \\ = \ & 6\,\tfrac{7}{12} \end{aligned}$$

<u>Example 3</u> Read through this example and the comment

$$4\frac{2}{3} - 1\frac{1}{2}$$

$$= 4 + \frac{2}{3} - (1 + \frac{1}{2})$$

$$= 4 - 1 + \frac{2}{3} - \frac{1}{2}$$

$$= 3 + \frac{4}{6} - \frac{3}{6}$$

Subtract the whole numbers and subtract the numbers named by the fractions. The L.C.D. is 6.

$$= 3 + \frac{1}{6}$$

$$= 3\frac{1}{6}$$

You can probably do an example like this without writing down steps 2, 3 or 5.

<u>Example 4</u> Read through this example and the comments

Sometimes in subtraction it is necessary to "borrow". This happens when the number to be subtracted is greater than the number it is to be subtracted from.

$$4\frac{1}{3} - 1\frac{3}{4}$$

$$= 3 + \frac{4}{12} - \frac{9}{12}$$

Subtract the whole numbers. When attempting to subtract the numbers named by fractions we see that $\frac{9}{12}$ is greater than $\frac{4}{12}$.

$$= 2 + 1\frac{4}{12} - \frac{9}{12}$$

"Borrow 1" from the whole number

$$= 2 + \frac{16}{12} - \frac{9}{12}$$

$1\frac{4}{12} = \frac{16}{12}$. Now we can subtract.

$$= 2 + \frac{7}{12}$$

$$= 2\frac{7}{12}$$

2. Subtract: $5\frac{3}{8} - 4\frac{5}{6}$

Answer: 2: $5\frac{3}{8} - 4\frac{5}{6}$

$$= 1 + \frac{9}{24} - \frac{20}{24}$$

$$= 1\frac{9}{24} - \frac{20}{24}$$

$$= \frac{33}{24} - \frac{20}{24}$$

$$= \frac{13}{24}$$

(addition may also be done with the numbers arranged vertically).

<u>Examples 5 and 6</u> Read through these examples.

$$\begin{array}{rcl} 3\frac{1}{4} & = & 3 + \frac{3}{12} \\ 2\frac{1}{3} & = & 2 + \frac{4}{12} \\ + \ 4\frac{1}{6} & = & 4 + \frac{2}{12} \\ \hline & & 9 + \frac{9}{12} \\ & = & 9\frac{3}{4} \end{array}$$

L.C.D. is 12

Basic fraction for $\frac{9}{12}$ is $\frac{3}{4}$

$$\begin{array}{rcl} 3\frac{3}{4} & = & 3 + \frac{18}{24} \\ \frac{5}{6} & = & \frac{20}{24} \\ + \ 1\frac{7}{8} & = & 1 + \frac{21}{24} \\ \hline & & 4 + \frac{59}{24} \\ & = & 4 + 2 + \frac{11}{24} \\ & = & 6\frac{11}{24} \end{array}$$

$\frac{59}{24} = 2\frac{11}{24}$

3. Add:

$$\begin{array}{r} \frac{9}{10} \\ 2\ \frac{1}{4} \\ + \ 1\ \frac{2}{5} \\ \hline \end{array}$$

<u>Example 7</u> Read through this example

$$\begin{array}{rclclcl} 12\frac{1}{3} & = & 12 + \frac{8}{24} & = & 11 + 1\frac{8}{24} & = & 11 + \frac{32}{24} \\ - \ 3\frac{5}{8} & = & 3 + \frac{15}{24} & = & 3 + \frac{15}{24} & = & 3 + \frac{15}{24} \\ \hline & & & & & & 8 + \frac{17}{24} \\ & & & & & = & 8\frac{17}{24} \end{array}$$

$\frac{15}{24}$ is greater than $\frac{8}{24}$.

We need to "borrow 1" from the 12.

4. Subtract

$$\begin{array}{r} 4\frac{1}{6} \\ - \ 3\frac{4}{5} \\ \hline \end{array}$$

Now read Objectives I3.1 and I3.2 and their examples. These tell you what you are expected to be able to do for this section. When ready, turn to and do CHECK EXERCISES 3.1 and 3.2.

Answers: 3:

$$\begin{array}{rcl} \frac{9}{10} & = & \frac{18}{20} \\ 2\frac{1}{4} & & 2 + \frac{5}{20} \\ + \ 1\frac{2}{5} & & 1 + \frac{8}{20} \\ \hline & & 3 + \frac{31}{20} \\ & = & 3 + 1 + \frac{11}{20} \\ & = & 4\frac{11}{20} \end{array}$$

4:

$$\begin{array}{rclclcl} 4\frac{1}{6} & = & 4 + \frac{5}{30} & = & 3 + 1\frac{5}{30} & = & 3 + \frac{35}{30} \\ - \ 3\frac{4}{5} & = & 3 + \frac{24}{30} & = & 3 + \frac{24}{30} & = & 3 + \frac{24}{30} \\ \hline & & & & & & \frac{11}{30} \end{array}$$

CHECK EXERCISE 3

I3.1 Find the sums and write each as a mixed numeral with fractional part as a basic fraction:

a) $3\frac{9}{10} + 2\frac{3}{8} + 1\frac{3}{4}$

b) $4\frac{2}{3} + 3\frac{1}{5} + 5\frac{7}{12}$

c) $4\frac{1}{3} + 2\frac{3}{10} + 2\frac{1}{2}$

d) $\begin{array}{r} 1\frac{3}{4} \\ 2\frac{5}{12} \\ + \frac{7}{2} \\ \hline \end{array}$

e) $\begin{array}{r} 9\frac{1}{5} \\ \frac{1}{3} \\ + 8 \\ \hline \end{array}$

I3.2 Find the differences and write the fractional parts as basic fractions:

a) $8\frac{5}{6} - 4\frac{3}{10}$

b) $7\frac{1}{6} - 4\frac{3}{4}$

c) $17\frac{5}{8} - 5\frac{1}{4}$

d) $\begin{array}{r} 91\frac{4}{7} \\ - 39\frac{7}{8} \\ \hline \end{array}$

e) $\begin{array}{r} 45\frac{5}{9} \\ - 37\frac{5}{6} \\ \hline \end{array}$

- Check your answers with those given at the end of the topic. You are successful in each CHECK EXERCISE if you are using the correct method and have no more than one part incorrect in each question.
- If you are not sure how to add mixed numerals or if you were unsuccessful with CHECK EXERCISE 3.1, read section 3 and do Activity exercises 3.1.
- If you are not sure how to subtract mixed numerals or if you were unsuccessful with CHECK EXERCISE 3.2 read section 3 and do Activity exercises 3.2.
- Otherwise go on to Section 4.

Activity Exercise 3.1

To add $4\frac{1}{5} + 3\frac{2}{5}$ we first write each mixed numeral as the sum

$4 + \frac{1}{5} + 3 + \frac{2}{5}$ of a whole number and a number named by a fraction

$4 + 3 + \frac{1}{5} + \frac{2}{5}$ We add the whole numbers and then the numbers named by the fractions.

$7 + \frac{3}{5}$ We write the result

$7\frac{3}{5}$

1. Add $5\frac{3}{7} + 3\frac{2}{7}$

Carefully read through the following examples and comments:

a) $*2\frac{3}{8} + 5\frac{3}{4} + 3\frac{5}{6}$

$= 2 + \frac{3}{8} + 5 + \frac{3}{4} + 3 + \frac{5}{6}$

$= 2 + 5 + 3 + \frac{3}{8} + \frac{3}{4} + \frac{5}{6}$ in order to add the numbers named by fractions we need to express them with their L.C.D. of 24.

$*= 10 + \frac{9}{24} + \frac{18}{24} + \frac{20}{24}$

$*= 10 + \frac{47}{24}$

$= 10 + 1\frac{23}{24}$ the mixed numeral for $\frac{47}{24}$ is $1\frac{23}{24}$

$= 10 + 1 + \frac{23}{24}$

$= 11 + \frac{23}{24}$

$*= 11\frac{23}{24}$ expressed as a mixed numeral

* When doing such an example you would probably only need to write down the lines marked with an *.

b) $* \quad 5\frac{5}{6} + 6\frac{1}{2}$

$= 11 + \frac{5}{6} + \frac{1}{2}$

$*= 11 + \frac{5}{6} + \frac{3}{6}$ L.C.D. is 6

$*= 11 + \frac{8}{6}$ $\frac{8}{6}$ is the mixed numeral $1\frac{2}{3}$

$= 11 + 1 + \frac{2}{6}$

$*= 12\frac{2}{6}$ $\frac{2}{6}$ is the basic fraction $\frac{1}{3}$

$*= 12\frac{1}{3}$

Note: Always express the mixed numeral with a BASIC FRACTION.

2. Add the following:

a) $3\frac{9}{10} + 4\frac{4}{5}$

b) $11\frac{7}{10} + 16\frac{3}{4} + 5\frac{1}{2}$

c) $7\frac{1}{4} + 3\frac{7}{8} + 4\frac{5}{6}$

d) $\begin{array}{r} 2\frac{1}{2} \\ \frac{3}{5} \\ +\ 1\frac{1}{2} \\ \hline \end{array}$

e) $\begin{array}{r} 2\frac{3}{5} \\ 3\frac{4}{7} \\ +\ 5\frac{3}{10} \\ \hline \end{array}$

Activity Exercise 3.2

$$4\frac{5}{7} - 2\frac{3}{7}$$

$= 4 + \frac{5}{7} - (2 + \frac{3}{7})$ subtract whole numbers and numbers

$= 4 - 2 + \frac{5}{7} - \frac{3}{7}$ named by fractions.

$= 2 + \frac{2}{7}$

$= 2\frac{2}{7}$ express as a mixed numeral

1. Find the difference:

$$8\frac{4}{5} - 5\frac{1}{5}$$

In order to subtract $7\frac{3}{4} - 5\frac{1}{8}$ we need to express the fractions with their L.C.D.

Thus $7\frac{3}{4} - 5\frac{1}{8}$ (You can do several steps at once.)

$= 7 - 5 + \frac{6}{8} - \frac{1}{8}$ fractions expressed with L.C.D. of 8.

$= 2 + \frac{5}{8}$

$= 2\frac{5}{8}$ expressed as a mixed numeral.

2. Find the difference:

$$5\frac{2}{3} - 2\frac{2}{5}$$

Here is another example:

$$\begin{array}{r} 27\frac{1}{3} \\ -18\frac{5}{6} \\ \hline \end{array}$$

$$\begin{array}{r} 27\frac{1}{3} \\ -\ 18\frac{5}{6} \\ \hline \end{array} \quad = \quad \begin{array}{r} 27 + \frac{2}{6} \\ -\ 18 + \frac{5}{6} \\ \hline \end{array}$$

We have expressed the fractions in the mixed numerals with common denominators. But we find we cannot subtract $\frac{5}{6}$ from $\frac{2}{6}$. It is too big. Thus we must "borrow" one from the whole number.

$$27 + \frac{2}{6} = 26 + 1 + \frac{2}{6}$$
$$= 26 + 1\frac{2}{6}$$
$$= 26 + \frac{8}{6}$$

Now we can subtract $\frac{5}{6}$ from $\frac{8}{6}$. We have

$$\begin{array}{rcrcr} 27\frac{1}{3} & = & 27 + \frac{2}{6} & = & 26 + \frac{8}{6} \\ -\ 18\frac{5}{6} & = & 18 + \frac{5}{6} & = & 18 + \frac{5}{6} \\ \hline & & & & 8 + \frac{3}{6} \\ & & & = & 8\frac{1}{2} \end{array}$$

a mixed numeral with a basic fraction.

3. Find the differences of the following and write the fractions as basic fractions.

a) $6\frac{1}{2} - 5\frac{1}{4}$

b) $4\frac{1}{4} - 3\frac{3}{16}$

c) $5\frac{1}{4} - 2\frac{3}{8}$

d) $\begin{array}{r} 4\frac{3}{16} \\ -\ 3\ \frac{1}{4} \\ \hline \end{array}$

e) $\begin{array}{r} 8\frac{3}{10} \\ -\ 5\ \frac{5}{7} \\ \hline \end{array}$

- Check your answers with those given at the end of the topic.
- Read objectives I3.1 and I3.2.
- If you feel you are ready, do CHECK EXERCISE 3A.

CHECK EXERCISE 3A

I3.1 Find the sums and write each as a mixed numeral with fraction as a basic fraction:

a) $3\frac{4}{5} + 7\frac{3}{10} + 2\frac{1}{2}$

b) $8\frac{2}{3} + \frac{7}{8} + 6\frac{1}{4}$

c) $3\frac{7}{9} + 5\frac{5}{6} + 2\frac{3}{4}$

d) $\begin{array}{r} 4\frac{3}{7} \\ 2\frac{5}{6} \\ +\ 5\frac{1}{4} \\ \hline \end{array}$

e) $\begin{array}{r} 1\frac{7}{15} \\ \frac{4}{5} \\ +\ 8\frac{5}{12} \\ \hline \end{array}$

I3.2 Find the differences and express the fractions as basic fractions:

a) $3\frac{1}{2} - 2\frac{1}{4}$

b) $4\frac{1}{3} - 2\frac{5}{12}$

c) $16\frac{1}{4} - 15\frac{3}{5}$

d) $\begin{array}{r} 2\ \frac{1}{7} \\ -\ 1\frac{9}{14} \\ \hline \end{array}$

e) $\begin{array}{r} 13\frac{5}{6} \\ -\ 12\frac{11}{12} \\ \hline \end{array}$

- Check your answers with those given at the end of the topic. You are successful if you use the correct method and you have no more than one part incorrect in each CHECK EXERCISE.
- If you are not sure how to add or subtract rational numbers named by mixed numerals, or if you were unsuccessful on either CHECK EXERCISE, check reference book <u>Modern School Mathematics,</u> pp 346, 347 or ask a student helper
  or ask your teacher.
- Otherwise, go on to section 4.

OBJECTIVE B4.1

To solve conditions for equality involving addition and subtraction of rational numbers.

Example

Solve each condition. The universe for the variables is the set of rational numbers.

A. $\frac{3}{5} + n = 2\frac{7}{10}$ B. $m - \frac{11}{3} = \frac{5}{2}$

Criterion: Method correct in each example and no more than one error in computation.

SOLUTION

A.

$$n = 2\frac{7}{10} - \frac{3}{5}$$
$$= 2 + \frac{7}{10} - \frac{6}{10}$$
$$= 2\ \frac{1}{10}$$

Check: $\frac{3}{5} + 2\frac{1}{10}$
$$= 2 + \frac{6}{10} + \frac{1}{10}$$
$$= 2\frac{7}{10}$$

Solution: $2\frac{1}{10}$

B.

$$m = \frac{5}{2} + \frac{11}{3}$$
$$= \frac{15}{6} + \frac{22}{6}$$
$$= \frac{37}{6}$$

Check: $\frac{37}{6} - \frac{11}{3}$
$$= \frac{37}{6} - \frac{22}{6}$$
$$= \frac{15}{6}$$
$$= \frac{5}{2}$$

Solution: $\frac{37}{6}$

OBJECTIVE I4.1

To solve conditions for equality involving addition and subtraction of rational numbers.

Example

Solve each condition. The universe for the variables is the set of rational numbers.

A. $3\frac{5}{6} = a + 1\frac{3}{5}$ *B.* $\frac{7}{8} = 2\frac{1}{7} - p$

Criterion: Method correct in each example and no more than one error in computation.

SOLUTION

A.

$$a = 3\tfrac{5}{6} - 1\tfrac{3}{5}$$
$$= 2 + \frac{25}{30} - \frac{18}{30}$$
$$= 2\tfrac{7}{30}$$

Check: $2\tfrac{7}{30} + 1\tfrac{3}{5}$

$$= 3 + \frac{7}{30} + \frac{18}{30}$$
$$= 3 + \frac{25}{30}$$
$$= 3\tfrac{5}{6}$$

Solution: $2\tfrac{7}{30}$

B.

$$p + \frac{7}{8} = 2\tfrac{1}{7}$$
$$p = 2\tfrac{1}{7} - \frac{7}{8}$$
$$= 2 + \frac{8}{56} - \frac{49}{56}$$
$$= 1 + 1\tfrac{8}{56} - \frac{49}{56}$$
$$= 1 + \frac{64}{56} - \frac{49}{56}$$
$$= 1\tfrac{15}{56}$$

Check: $1\tfrac{15}{56} + \frac{7}{8}$

$$= 1 + \frac{15}{56} + \frac{49}{56}$$
$$= 1 + \frac{64}{56}$$
$$= 1 + 1\tfrac{8}{56}$$
$$= 2\tfrac{1}{7}$$

Solution: $1\tfrac{15}{56}$

Section 4. Solving conditions for equality with rational numbers.

You have already solved conditions for equality in which the universe or replacement set for the variables is the set of whole numbers.

e.g. $x + 7 = 15$

The solution is the number which when put in place of x makes the condition a true statement. The solution is 8 in this case.

Now we are going to solve conditions for equality in which the universe or replacement set for the variables is the set of rational numbers. e.g. $\frac{3}{4} + n = \frac{7}{4}$

To solve these conditions we will make use of the following ideas:

Conditions involving addition have related conditions involving subtraction	AND	Conditions involving subtraction have related conditions involving addition
$n + \frac{3}{5} = \frac{7}{5}$		$a - \frac{3}{5} = \frac{1}{5}$
$n = \frac{7}{5} - \frac{3}{5}$ Related subtraction condition with n alone on one side		$a = \frac{1}{5} + \frac{3}{5}$ Related addition condition with a alone on one side

1. *Find two related subtraction conditions for* $\frac{3}{2} + x = \frac{7}{4}$
2. *Find a related addition condition with the variable alone on one side for* $3\frac{3}{4} = y - 2\frac{1}{2}$
3. *Find a related subtraction condition with the variable alone on one side for* $\frac{1}{2} - s = \frac{1}{3}$

Answers: 1: $x = \frac{7}{4} - \frac{3}{2}$; $\frac{3}{2} = \frac{7}{4} - x$

2: $y = 3\frac{3}{4} + 2\frac{1}{2}$

3: $\left.\begin{array}{l} \frac{1}{2} = \frac{1}{3} + s \\ s = \frac{1}{2} - \frac{1}{3} \end{array}\right\}$ *two steps are needed*

Examples 1 and 2   Read through these examples and the comments

Solve: $1\frac{2}{3} + m = 4\frac{3}{4}$

$m = 4\frac{3}{4} - 1\frac{2}{3}$ Related condition with m alone on one side

$= 3 + \frac{9}{12} - \frac{8}{12}$ Do the subtraction

$= 3\frac{1}{12}$

Check:

$$\left.\begin{aligned} & 1\frac{2}{3} + 3\frac{1}{12} \\ &= 1\frac{8}{12} + 3\frac{1}{12} \\ &= 4\frac{9}{12} \text{ or } 4\frac{3}{4} \end{aligned}\right\}$$

Test that $3\frac{1}{12}$ in place of m makes the condition a true statement.

Solution: $3\frac{1}{12}$

Solve: $1\frac{5}{8} - b = \frac{3}{4}$

$1\frac{5}{8} = \frac{3}{4} + b$ *Related condition, but b is not alone on one side.*

$b = 1\frac{5}{8} - \frac{3}{4}$ *Related condition with b alone on one side.*

$= 1 + \frac{5}{8} - \frac{6}{8}$ *(Two steps were needed to get the related condition with b alone on one side.)*

$= 1\frac{5}{8} - \frac{6}{8}$

$= \frac{13}{8} - \frac{6}{8}$

$= \frac{7}{8}$

Check: $1\frac{5}{8} - \frac{7}{8}$ *Test that $\frac{7}{8}$ in place of b makes the original condition a true statement.*

$= \frac{13}{8} - \frac{7}{8}$

$= \frac{6}{8}$ *or* $\frac{3}{4}$

Solution: $\frac{7}{8}$

3. Solve $p - 3\frac{7}{10} = 8\frac{1}{6}$.

4. *Solve* $12 = 13\frac{7}{9} - x$

Now read OBJECTIVE I4.1 and its example. This tells you what you are expected to be able to do for this section.
When ready, turn to and do CHECK EXERCISE 4.1.

Answers:

3.
$$\begin{aligned} p &= 8\frac{1}{6} + 3\,\frac{7}{10} \\ &= 11 + \frac{5}{30} + \frac{21}{30} \\ &= 11\frac{26}{30} \\ &= 11\frac{13}{15} \end{aligned}$$

Check: $11\frac{13}{15} - 3\frac{7}{10}$
$$\begin{aligned} &= 8 + \frac{26}{30} - \frac{21}{30} \\ &= 8\frac{5}{30} \\ &= 8\frac{1}{6} \end{aligned}$$

Solution: $11\frac{13}{15}$

4.
$$\begin{aligned} 12 + x &= 13\frac{7}{9} \\ x &= 13\frac{7}{9} - 12 \\ &= 1\frac{7}{9} \end{aligned}$$

Check: $12 + 1\frac{7}{9} = 13\frac{7}{9}$

Solution: $1\frac{7}{9}$

CHECK EXERCISE 4

I4.1 Solve each condition. Universe for variables is the set of rational numbers. Write fractions in solutions as basic fractions.

a) $\frac{10}{12} = p + \frac{12}{15}$

b) $\frac{15}{16} - a = \frac{5}{6}$

c) $12\frac{12}{9} = 13 + n$

d) $16\frac{1}{3} - 3\frac{7}{8} = b$

e) $8\frac{1}{5} - a = \frac{3}{7}$

- Check your answers with those given at the end of the topic.
- If you are not sure how to solve these conditions or if you had more than one part incorrect, read objective 4 carefully and do activity exercise 4.1.
- Otherwise, go to section 5.

Activity Exercise 4.1

For all the variables in this section, the universe is the set of rational numbers.

For a condition of equality involving addition, there is a related condition involving subtraction; and for a condition for equality involving subtraction; there is a related condition involving addition.

Carefully read the following example and comments:

$1\frac{3}{4} + a = 3\frac{1}{2}$

$a = 3\frac{1}{2} - 1\frac{3}{4}$ — related condition involving subtraction with 'a' alone on one side.

$a = 2 + \frac{2}{4} - \frac{3}{4}$ — subtracting whole numbers and numbers named by fractions.

$= 1 + 1\frac{2}{4} - \frac{3}{4}$ — borrowing 1 from the 2.

$= 1 + \frac{6}{4} - \frac{3}{4}$

$= 1 + \frac{3}{4}$

$= 1\frac{3}{4}$ — mixed numeral.

We must check the answer. Does $1\frac{3}{4} + 1\frac{3}{4} = 3\frac{1}{2}$?

Check: $1\frac{3}{4} + 1\frac{3}{4} = 2 + \frac{3}{4} + \frac{3}{4}$

$= 2 + \frac{6}{4}$

$= 2 + 1\frac{2}{4}$

$= 3\frac{1}{2}$

Solution: $1\frac{3}{4}$

1. Solve the following conditions. Check them also.

a) $2\frac{3}{5} + n = 5\frac{4}{5}$

b) $b + 4\frac{4}{7} = 6\frac{3}{5}$

Carefully read these examples and comments:

a) $n - \frac{3}{4} = 1\frac{2}{3}$

$n = 1\frac{2}{3} + \frac{3}{4}$ — related condition involving addition with 'n' alone on one side.

$n = 1 + \frac{2}{3} + \frac{3}{4}$

$= 1 + \frac{8}{12} + \frac{9}{12}$ — L.C.D. is 12.

$= 1 + \frac{17}{12}$

$= 1 + 1\frac{5}{12}$ — change to a mixed numeral.

$= 2\frac{5}{12}$

Check:

$2\frac{5}{12} - \frac{3}{4}$

$= 2 + \frac{5}{12} - \frac{9}{12}$ — L.C.D. is 12

$= 1 + 1\frac{5}{12} - \frac{9}{12}$ — must 'borrow' one to subtract.

$= 1 + \frac{17}{12} - \frac{9}{12}$

$= 1 + \frac{8}{12}$

$= 1\frac{8}{12}$

$= 1\frac{2}{3}$ — change to a basic fraction.

Solution: $2\frac{5}{12}$

b) $4\frac{1}{3} - b = 2\frac{3}{4}$

$4\frac{1}{3} = 2\frac{3}{4} + b$ — related condition involving addition. b is NOT alone on one side - another step is needed.

$b = 4\frac{1}{3} - 2\frac{3}{4}$ — related condition involving subtraction now has b alone on one side.

$b = 2 + \frac{1}{3} - \frac{3}{4}$ — subtract whole numbers and numbers named by fractions.

$= 2 + \frac{4}{12} - \frac{9}{12}$ — L.C.D. is 12.

$= 1 + 1\frac{4}{12} - \frac{9}{12}$ — must 'borrow' one.

$= 1 + \frac{16}{12} - \frac{9}{12}$

$= 1\frac{7}{12}$

Check: $4\frac{1}{3} - 1\frac{7}{12}$

$= 3 + \frac{4}{12} - \frac{7}{12}$ — L.C.D. is 12.

$= 2 + 1\frac{4}{12} - \frac{7}{12}$ — need to 'borrow' one to subtract.

$= 2 + \frac{16}{12} - \frac{7}{12}$

$= 2\frac{9}{12}$

$= 2\frac{3}{4}$ — mixed numeral with a basic fraction.

Solution: $1\frac{7}{12}$

2. Solve the following conditions and check the solutions.

a) $a - 4\frac{2}{5} = 1\frac{1}{3}$ b) $5\frac{4}{9} - b = 3\frac{2}{3}$

- Check your answers with those given at the end of the topic.
- Read objective I4.1.
- If you are ready, do CHECK EXERCISE 4A.

CHECK EXERCISE 4A

I4.1 Solve each condition. The universe for variables is the set of rational numbers. Write fractions in solutions as basic fractions.

a) $11\frac{3}{4} - r = 10\frac{5}{8}$

b) $8\frac{1}{2} + b = 12\frac{5}{6}$

c) $y + 14\frac{3}{5} = 21\frac{1}{4}$

d) $a - 6\frac{4}{5} = 8\frac{1}{2}$

e) $10\frac{9}{16} - n = 5\frac{3}{8}$

- Check your answers with those given at the end of the topic.
- If you do not feel sure about solving conditions, or if you had more than one part incorrect ask a student helper or ask your teacher.
- Otherwise, go on to section 5.

Summary

Section 1

Addition and subtraction of rational numbers named by fractions is usually done by obtaining equivalent fractions with least common denominators and then adding or subtracting the numerators. Answers are given as basic fractions.

Section 2

A mixed numeral contains a whole number numeral and a fraction.

Each mixed numeral has a corresponding fraction

$$4\frac{3}{8} = (4 \times 8) + 3 \text{ eighths}$$
$$= 35 \text{ eighths}$$
$$= \frac{35}{8}$$

AND

Each fraction with numerator greater than denominator has a corresponding mixed numeral.

$$\frac{35}{8} = 8\overline{)35} = 4\frac{3}{8}$$

Section 3

In addition (or subtraction) of rational numbers named by mixed numerals, the whole numbers are added (or subtracted) and the numbers named by the fractions are added (or subtracted).

Section 4

A solution for a condition for equality is a number, which when put in place of the variable, makes the condition a true statement.

Conditions for equality involving addition (or subtraction) of rational numbers are solved by finding (related conditions with the variable alone on one side.

Section 5

Addition and subtraction of rational numbers are used to answer questions about every day situations. Most examples are done by

(1) finding the relationship between what is asked and what is given.

(2) obtaining the relationship in mathematical form and doing the indicated mathematical work to get a mathematical answer.

(3) Using the mathematical answer to answer the original question.

Section 6

$$\frac{a}{b} \text{ of } \frac{c}{d} = \frac{a \times c}{b \times d}$$

Section 7

Multiplication of rational numbers is done as follows:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

When mixed numerals or whole numbers are involved, their corresponding fractions are used. Answers are given as basic fractions. Reduction to basic fractions is done by dividing numerator and denominator by their common factors (cancelling).

Section 8

The reciprocal of a number is another number whose product with the given number is 1.

The reciprocal of a non-zero rational number is obtained by interchanging the numerator and denominator of the fraction for the rational number.

Zero has no reciprocal.

Section 9

To divide by a non-zero rational number, you multiply by its reciprocal.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

It is not possible to divide by 0.

Section 10

Conditions for equality involving multiplication of rational numbers can often be solved by finding related conditions involving division of rational numbers with the variable alone on one side.

Also, in conditions for equality in which the variable is multiplied by a number, the variable can be obtained alone on one side by multiplying both sides by the reciprocal of the number by which the variable is multiplied.

Section 11

To answer questions from everyday situations by mathematics we usually find the relationship between what is asked for and what is given. The mathematical form of this relationship is called the mathematical model for the situation. The mathematical model is often a condition for equality. The solution for this condition for equality is used to answer the original question.

Section 12

The rational numbers are closed for addition and multiplication. Addition and multiplication of rational numbers are commutative and associative.

0 and 1 are the identities for addition and multiplication respectively of rational numbers.

Multiplication of rational numbers is distributive over addition. Non-zero rational numbers have reciprocals. This is a property of rational numbers which whole numbers do not have.

Vocabulary

analyse - with respect to a question about an every day situation it means to find the relationship between what is asked for and what is given. (page) (section 5, p 2)

application - use of mathematics to answer a question about an everyday situation. (page) (section 5,p 1)

associative - the property of an operation which permits a change in the grouping of the numbers to not affect the result (page) (section 12,p 1)

binary - a binary operation is done with just two numbers at a time. (page) (section 12, p 4)

cancel - to divide numerator and denominator of a fraction by a common factor. (page) (section 7, p 3)

closed - the property of an operation which states that the number resulting from the operation is in the same set as the numbers the operation is done with. (page) (section 12, p 1)

commutative - the property of an operation which permits a change in the order of the numbers to not affect the result. (page) (section 12, p 1)

complex fraction - a fraction like $\frac{\frac{2}{3}}{\frac{3}{4}}$ which represents $\frac{2}{3} \div \frac{3}{4}$.

condition for equality - a mathematical statement involving a variable and the = symbol. (page) (section 4, p 1)

difference - the result of a subtraction. (page) (section 1, Objectives)

distributive - a property of multiplication and addition of numbers (page) (section 12, p 1)

L.C.D. - least common denominator. (page) (section 1, p 2)

mixed numeral - a numeral involving a whole number numeral and a fraction. (page) (section 2, p 1)

model - the mathematical form for a relationship. (page) (section 11, p 1)

operation - addition, subtraction, multiplication and division are examples of operations. (page) (Introduction)

product - the result of a multiplication. (page) (section 7, p 1)

property - a characteristic or quality. (page) (section 12, p 1)

quotient - the result of a division.

reciprocal - a number whose product with a given number is 1. (page) (section 8, p 1)

related condition - a condition with the same solution as the given condition. (page) (section 4, p 1)

replacement set - the set of numbers which a variable can represent; same as universe. (page) (section 4, p 1)

solution - a number which makes a condition a true statement. (page) (section 4, p 1)

solve - the process of finding the solution to a condition or problem. (page) (section 4, p 1)

sum - the result of an addition. (page) (section 4, p 1)

universe - the set of numbers which a variable can represent; same as replacement set. (page) (section 4, p 1)

variable - a letter which can represent any of the numbers in a given set. (page) (section 4, p 1)

Review Exercises Topic II

Operations with rational numbers

You have completed the study of Topic II. It would be nice to know just how much you have learned in this topic. To do this you take a test, but before you take the test you should carefully review what you were expected to learn. To help you with this review we have prepared a set of exercises. If you have difficulty with any one of the exercises you should go back and review the appropriate objective (pink pages) and its development (white pages). The appropriate objective(s) for each exercise is (are) shown in the left margin next to the exercise.

I1.1 Find the sums and write each as a basic fraction:

A. $\frac{7}{9} + \frac{2}{5} + \frac{2}{3}$

B. $\begin{array}{r} \frac{7}{10} \\ \frac{1}{5} \\ \frac{5}{12} \\ +\ \frac{1}{2} \\ \hline \end{array}$

I1.2 Find the differences for the following and write each as a basic fraction:

A. $\frac{9}{10} - \frac{3}{4}$

B. $\begin{array}{r} \frac{5}{12} \\ -\ \frac{3}{10} \\ \hline \end{array}$

I2.1 A. Write a mixed numeral for each of the following fractions

i) $\frac{83}{6}$ ii) $\frac{257}{8}$

B. Write the following mixed numerals as fractions:

i) $6\frac{3}{5}$ ii) $15\frac{3}{8}$

I2.2 A. Justify that the mixed numeral for $\frac{43}{8}$ is $5\frac{3}{8}$.

B. Justify that the fraction for $7\frac{4}{5}$ is $\frac{39}{5}$.

I3.1 Find the following sums and write the fractional part as a basic fraction:

A. $3\frac{7}{8} + 2\frac{3}{4} + 1\frac{1}{2} + 6\frac{1}{4}$

B. $$\begin{array}{r} 32\frac{3}{8} \\ 16\frac{1}{3} \\ +\ 15\frac{5}{6} \\ \hline \end{array}$$

I3.2 Find the following differences and write the fractional part as a basic fraction:

A. $$\begin{array}{r} 8\frac{4}{7} \\ -\ 3\frac{5}{8} \\ \hline \end{array}$$

B. $4\frac{5}{12} - \frac{7}{9}$

I4.1 Solve each condition and write the fractions in the solutions as basic fractions:

A. $3\frac{5}{7} = n + 1\frac{3}{5}$

B. $n - \frac{3}{10} = 1\frac{5}{8}$

I5.1 John went on a hike. He covered a total distance of $2\frac{3}{16}$ miles. Yet when he came home he declared that he had walked only $1\frac{2}{3}$ miles. When questioned about this statement he said that he had run $\frac{25}{48}$ mile. Was John correct when he said that he had walked $1\frac{2}{3}$ miles?

I6.1 Show by diagrams how you can obtain $\frac{3}{4}$ of $\frac{4}{5}$.

I7.1 Find the basic fraction for each product:

A. $\frac{49}{66} \times \frac{27}{35} \times \frac{55}{63}$

B. $3 \times \frac{14}{75} \times 6\frac{3}{7}$

I8.1 i) Give an example that illustrates the property which a number and its reciprocal have.

ii) What is the reciprocal of 1?

iii) Write a whole number other than one and give its reciprocal.

iv) Give the rational number that has no reciprocal and explain why it has no reciprocal.

I9.1 Find the quotients of the following pairs of rational numbers:

A. $7\frac{5}{7} \div 3\frac{3}{14}$

B. $$\frac{2\frac{1}{4}}{3\frac{3}{5}}$$

I9.2 Explain in terms of reciprocals why $\frac{7}{12} \div 0$ is not possible.

I10.1 Solve each of the following conditions:

A. $6\frac{1}{4}\,a = 5\frac{5}{8}$ B. $1\frac{3}{4} = 2\frac{3}{16}\,n$

I11.1 A. A dealer bought a motor bike for $1200. He added $\frac{1}{6}$ of this cost to determine his selling price. What was his selling price?

B. Two sides of a triangular flower bed are $1\frac{2}{3}$ yards and $2\frac{3}{4}$ yards long. The perimeter of the flower bed is 8 yards. What is the length of the third side?

C. An inch is about $2\frac{1}{2}$ centimeters. How many inches are there in $26\frac{1}{4}$ centimeters?

I12.1 Give a numerical example which illustrates what is meant by each of the following statements. (one example for each)

(a) the set of rational numbers is closed under addition.

(b) addition of rational numbers is commutative.

(c) addition of rational numbers is associative.

(d) the set of rational numbers has an identity element for addition.

I12.2 Show what is meant by each of the following statements by giving a numerical example for each.

(a) the set of rational numbers is closed under multiplication.

(b) multiplication of rational numbers is commutative.

(c) multiplication of rational numbers is associative.

(d) the set of rational numbers has an identity element for multiplication.

(e) rational numbers have the property that multiplication is distributive over addition.

(f) every non-zero rational number has a reciprocal.

I12.3 (a) State a property of multiplication of rational numbers which the whole numbers do not have for multiplication.

(b) Give an example to illustrate this property of the rational numbers and one to show that the whole numbers do not have it.

Solutions - Topic 2 Phase I

CHECK EXERCISE 1

I1.1 a) $\frac{67}{50}$ b) $\frac{95}{48}$ c) $\frac{31}{15}$ d) $\frac{41}{20}$ e) $\frac{77}{30}$

I1.2 a) $\frac{1}{2}$ b) $\frac{3}{10}$ c) $\frac{5}{18}$ d) $\frac{27}{40}$ e) $\frac{11}{12}$

Activity exercise 1.1

1 a) $\frac{6}{7}$ b)i) $\frac{7}{9}$ ii) $\frac{7}{4}$ iii) $\frac{11}{10}$ iv) $\frac{4}{5}$ c)i) $\frac{5}{4}$ ii) $\frac{5}{6}$ iii) $\frac{44}{15}$
iv) $\frac{71}{63}$ v) $\frac{187}{70}$ d)i) $\frac{2}{3}$ ii) $\frac{9}{10}$ iii) $\frac{47}{40}$ e)i) 2 ii) $\frac{9}{8}$
iii) $\frac{53}{40}$ iv) $\frac{23}{15}$ v) $\frac{9}{4}$

Activity exercise 1.2

a)i) $\frac{1}{9}$ ii) $\frac{4}{5}$ iii) $\frac{2}{8}$ b)i) $\frac{25}{15}$ ii) $\frac{7}{24}$ iii) $\frac{11}{35}$
c)i) $\frac{3}{16}$ ii) $\frac{1}{4}$ iii) $\frac{1}{4}$ iv) $\frac{1}{12}$ v) $\frac{43}{35}$

CHECK EXERCISE 1A

I1.1 a) $\frac{8}{9}$ b) $\frac{71}{100}$ c) $\frac{7}{12}$ d) $\frac{5}{6}$ e) $\frac{23}{24}$

I1.2 a) $\frac{1}{3}$ b) $\frac{3}{10}$ c) $\frac{7}{24}$ d) $\frac{5}{48}$ e) $\frac{3}{10}$

CHECK EXERCISE 2

B2.1 i) a) $7\frac{2}{5}$ b) $2\frac{1}{4}$ c) $3\frac{2}{3}$ d) $1\frac{6}{7}$ e) $1\frac{7}{9}$

ii) a) $\frac{7}{4}$ b) $\frac{19}{7}$ c) $\frac{19}{11}$ d) $\frac{40}{9}$ e) $\frac{43}{8}$

I2.2 i) a)
$$\begin{aligned}\frac{15}{4} &= \frac{12}{4} + \frac{3}{4}\\ &= \frac{12 \div 4}{4 \div 4} + \frac{3}{4}\\ &= \frac{3}{1} + \frac{3}{4}\\ &= 3 + \frac{3}{4}\\ &= 3\frac{3}{4}\end{aligned}$$

b)
$$\begin{aligned}\frac{22}{7} &= \frac{21}{7} + \frac{1}{7}\\ &= \frac{21 \div 7}{7 \div 7} + \frac{1}{7}\\ &= \frac{3}{1} + \frac{1}{7}\\ &= 3 + \frac{1}{7}\\ &= 3\frac{1}{7}\end{aligned}$$

2 3 1

I2.2ii) a) $3\frac{3}{5} = 3 + \frac{3}{5}$
$= \frac{3}{1} + \frac{3}{5}$
$= \frac{3 \times 5}{1 \times 5} + \frac{3}{5}$
$= \frac{15}{5} + \frac{3}{5}$
$= \frac{18}{5}$

b) $2\frac{1}{4} = 2 + \frac{1}{4}$
$= \frac{2}{1} + \frac{1}{4}$
$= \frac{2 \times 4}{1 \times 4} + \frac{1}{4}$
$= \frac{8}{4} + \frac{1}{4}$
$= \frac{9}{4}$

Activity Exercise 2.1

1.a) $1\frac{2}{5}$ b) $2\frac{3}{5}$ c) $3\frac{1}{5}$ 2a) $3\frac{3}{4}$ b) $5\frac{2}{3}$ c) $2\frac{5}{8}$ d) $5\frac{1}{6}$ e) $3\frac{3}{7}$
3.a) $\frac{11}{4}$ 4.a) $\frac{27}{5}$ b) $\frac{11}{3}$ c) $\frac{11}{7}$ d) $\frac{21}{4}$ e) $\frac{25}{9}$

Activity Exercise 2.2.

1. $\frac{23}{5} = \frac{20}{5} + \frac{3}{5}$
$= \frac{20 \div 5}{5 \div 5} + \frac{3}{5}$
$= \frac{4}{1} + \frac{3}{5}$
$= 4 + \frac{3}{5}$
$= 4\frac{3}{5}$

2.a) $\frac{37}{6} = \frac{36}{6} + \frac{1}{6}$
$= \frac{36 \div 6}{6 \div 6} + \frac{1}{6}$
$= \frac{6}{1} + \frac{1}{6}$
$= 6 + \frac{1}{6}$
$= 6\frac{1}{6}$

2.b) $\frac{17}{3} = \frac{15}{3} + \frac{2}{3}$
$= \frac{15 \div 3}{3 \div 3} + \frac{2}{3}$
$= \frac{5}{1} + \frac{2}{3}$
$= 5 + \frac{2}{3}$
$= 5\frac{2}{3}$

3. $3\frac{5}{7} = 3 + \frac{5}{7}$
$= \frac{3}{1} + \frac{5}{7}$
$= \frac{3 \times 7}{1 \times 7} + \frac{5}{7}$
$= \frac{21}{7} + \frac{5}{7}$
$= \frac{26}{7}$

4.a) $2\frac{3}{4} = 2 + \frac{3}{4}$
$= \frac{2}{1} + \frac{3}{4}$
$= \frac{2 \times 4}{1 \times 4} + \frac{3}{4}$
$= \frac{8}{4} + \frac{3}{4}$
$= \frac{11}{4}$

b) $6\frac{5}{8} = 6 + \frac{5}{8}$
$= \frac{6}{1} + \frac{5}{8}$
$= \frac{6 \times 8}{1 \times 8} + \frac{5}{8}$
$= \frac{48}{8} + \frac{5}{8}$
$= \frac{53}{8}$

CHECK EXERCISE 2A

B2.1 i) a) $2\frac{4}{15}$ b) $2\frac{14}{17}$ c) $2\frac{7}{10}$ d) $1\frac{3}{7}$ e) $3\frac{2}{5}$

ii) a) $\frac{29}{8}$ b) $\frac{47}{6}$ c) $\frac{52}{15}$ d) $\frac{23}{12}$ e) $\frac{125}{22}$

I2.2 i) a) $\frac{28}{3} = \frac{27}{3} + \frac{1}{3} = \frac{27 \div 3}{3 \div 3} + \frac{1}{3} = \frac{9}{1} + \frac{1}{3} = 9 + \frac{1}{3} = 9\frac{1}{3}$

b) $\frac{20}{9} = \frac{18}{9} + \frac{2}{9} = \frac{18 \div 9}{9 \div 9} + \frac{2}{9} = \frac{2}{1} + \frac{2}{9} = 2 + \frac{2}{9} = 2\frac{2}{9}$

ii) a) $8\frac{5}{6} = 8 + \frac{5}{6} = \frac{8}{1} + \frac{5}{6} = \frac{8 \times 6}{1 \times 6} + \frac{5}{6} = \frac{48}{6} + \frac{5}{6} = \frac{53}{6}$

b) $7\frac{3}{4} = 7 + \frac{3}{4} = \frac{7}{1} + \frac{3}{4} = \frac{7 \times 4}{1 \times 4} + \frac{3}{4} = \frac{28}{4} + \frac{3}{4} = \frac{31}{4}$

CHECK EXERCISE 3

I3.1 a) $8\frac{1}{40}$ b) $13\frac{9}{20}$ c) $9\frac{2}{15}$ d) $7\frac{2}{3}$ e) $17\frac{8}{15}$

I3.2 a) $4\frac{8}{15}$ b) $2\frac{5}{12}$ c) $12\frac{3}{8}$ d) $51\frac{39}{56}$ e) $7\frac{13}{18}$

Activity Exercise 3.1

1. $8\frac{5}{7}$ 2a) $8\frac{7}{10}$ b) $33\frac{19}{20}$ c) $15\frac{23}{24}$ d) $4\frac{3}{5}$ e) $11\frac{33}{70}$

Activity Exercise 3.2

1. $3\frac{3}{5}$ 2. $3\frac{4}{15}$ 3.a) $1\frac{1}{4}$ b) $1\frac{1}{16}$ c) $2\frac{7}{8}$ d) $\frac{15}{16}$ e) $2\frac{41}{70}$

CHECK EXERCISE 3A

I3.1 a) $13\frac{3}{5}$ b) $15\frac{19}{24}$ c) $12\frac{13}{36}$ d) $12\frac{43}{84}$ e) $10\frac{41}{60}$

I3.2 a) $1\frac{1}{4}$ b) $1\frac{11}{12}$ c) $\frac{13}{20}$ d) $\frac{1}{2}$ e) $\frac{11}{12}$

CHECK EXERCISE 4

I4.1 a) $p = \frac{1}{30}$ b) $a = \frac{5}{48}$ c) $n = \frac{1}{3}$ d) $b = 12\frac{11}{24}$ e) $a = 7\frac{27}{35}$

Activity Exercise 4.1

1.a) $n = 3\frac{1}{5}$ b) $b = 2\frac{1}{35}$ 2a) $a = 5\frac{11}{15}$ b) $b = 1\frac{7}{9}$

CHECK EXERCISE 4A

I4.1 a) $r = 1\frac{1}{8}$ b) $b = 4\frac{1}{3}$ c) $y = 6\frac{13}{20}$ d) $a = 15\frac{3}{10}$ e) $n = 5\frac{3}{16}$

CHECK EXERCISE 5

I5.1 1. 2 miles 2. $35\frac{7}{8}$ miles 3. $1\frac{11}{12}$ feet 4. $3\frac{1}{3}$ hours

Activity Exercise 5

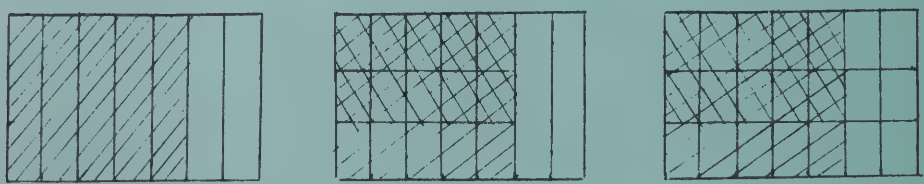
1. $\frac{103}{120}$ 2. $12\frac{1}{16}$ inches 3. $6\frac{1}{16}$ inches 4. $4\frac{85}{128}$ inches

CHECK EXERCISE 5A

I5.1 1. $1\frac{1}{30}$ hours 2. $12\frac{1}{2}$ inches 3. $7\frac{15}{16}$ inches 4. up $2\frac{13}{24}$ points

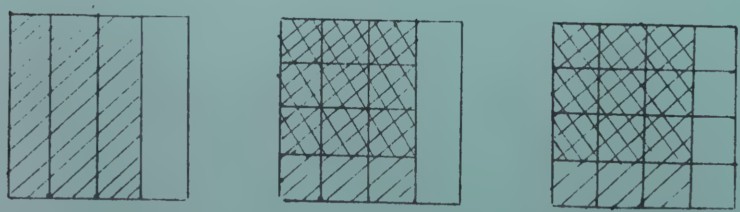
CHECK EXERCISE 6

I6.1 i)



$\frac{5}{7}$ of region $\frac{2}{3}$ of $\frac{5}{7}$ $\frac{2}{3}$ of $\frac{5}{7}$ is $\frac{10}{21}$

ii)



$\frac{3}{4}$ of region $\frac{3}{4}$ of $\frac{3}{4}$ $\frac{3}{4}$ of $\frac{3}{4}$ is $\frac{9}{16}$

Answers to Review Exercises Topic II

I1.1 A. $\frac{83}{45}$ B. $\frac{109}{60}$

I1.2 A. $\frac{3}{20}$ B. $\frac{7}{60}$

B2.1 A. i) $13\frac{5}{6}$; ii) $32\frac{1}{8}$ B. i) $\frac{33}{5}$; ii) $\frac{123}{8}$

I2.2 A. $\frac{43}{8} = \frac{40}{8} + \frac{3}{8}$

$= \frac{40 \div 8}{8 \div 8} + \frac{3}{8}$

$= \frac{5}{1} + \frac{3}{8}$

$= 5 + \frac{3}{8}$

$= 5\frac{3}{8}$

B. $7\frac{4}{5} = 7 + \frac{4}{5}$

$= \frac{7}{1} + \frac{4}{5}$

$= \frac{7 \times 5}{1 \times 5} + \frac{4}{5}$

$= \frac{35}{5} + \frac{4}{5}$

$= \frac{39}{5}$

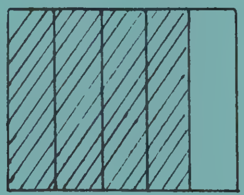
I3.1 A. $14\frac{3}{8}$ B. $64\frac{13}{24}$

I3.2 A. $4\frac{53}{56}$ B. $3\frac{23}{36}$

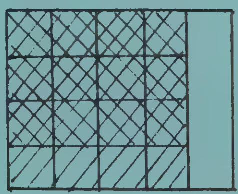
I4.1 A. $n = 2\frac{4}{35}$ B. $n = 1\frac{37}{40}$

I5.1 Yes

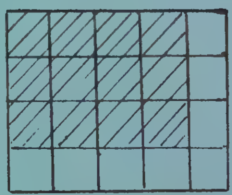
I6.1



$\frac{4}{5}$



$\frac{3}{4}$ of $\frac{4}{5}$



$\frac{3}{4}$ of $\frac{4}{5} = \frac{12}{20}$

I7.1 A. $\frac{1}{2}$ B. $\frac{18}{5}$ or $3\frac{3}{5}$

I8.1 i) $\frac{3}{4} \times \frac{4}{3} = 1$ any fraction can be used to illustrate this property

ii) 1

iii) can be any whole number, i.e. $17, \frac{1}{17}$.

iv) 0 has no reciprocal; $0 = \frac{0}{1}$ and there is no number by which $\frac{0}{1}$ can be multiplied to give 1, for $\frac{0}{1}$ times any number is 0.

OR. The reciprocal of $\frac{0}{1}$ is $\frac{1}{0}$, but $\frac{1}{0}$ has no meaning since we cannot divide by 0.

I9.1 A. $\frac{12}{5} = 2\frac{2}{5}$ B. $\frac{5}{8}$

I9.2 $\frac{7}{12} \div 0$ can be solved by converting to the corresponding multiplication and get $\frac{7}{12} \times \frac{1}{0}$, but $\frac{1}{0}$ does not exist and hence we cannot get the multiplication example. Therefore the division is not possible.

I10.1 A. $a = \frac{9}{10}$ B. $n = \frac{4}{5}$

I11.1 A. selling price is \$1400 ; B. $3\frac{7}{12}$ yards; C. $10\frac{1}{2}$ inches

I12.1 Examples which show the same ideas as:

a) $\frac{3}{5} + \frac{5}{7} = \frac{46}{35}$ and $\frac{46}{35}$ is a rational number.

b) $\frac{3}{5} + \frac{5}{7} = \frac{5}{7} + \frac{3}{5}$

c) $(\frac{3}{5} + \frac{5}{7}) + \frac{7}{8} = \frac{3}{5} + (\frac{5}{7} + \frac{7}{8})$

d) $\frac{3}{5} + 0 = \frac{3}{5}$

I12.2 Examples which show the same ideas as:

a) $\frac{3}{5} \times \frac{5}{7} = \frac{3}{7}$ and $\frac{3}{7}$ is a rational number.

b) $\frac{3}{5} \times \frac{5}{7} = \frac{5}{7} \times \frac{3}{5}$

c) $(\frac{3}{5} \times \frac{5}{7}) \times \frac{7}{9} = \frac{3}{5} \times (\frac{5}{7} \times \frac{7}{9})$

d) $\frac{3}{5} \times 1 = \frac{3}{5}$

e) $\frac{3}{5} \times (\frac{5}{7} + \frac{7}{9}) = (\frac{3}{5} \times \frac{5}{7}) + (\frac{3}{5} \times \frac{7}{9})$

f) reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.

I12.3 a) A statement which gives the same idea as: Every non-zero rational number has a reciprocal.

b) Examples which show the same idea as:

$\frac{3}{5}$ and $\frac{5}{3}$ are reciprocals for $\frac{3}{5} \times \frac{5}{3} = 1$.

3 has no reciprocal for there exists no whole number by which you can multiply 3 and get one as an answer.

Topic II - Operations With Rational Numbers

Post - Test I (Form C)

1. Show all your work in the spaces provided for this purpose.
2. Place your answers or solutions on the lines (_______) provided for this purpose on the right hand side of the page.
3. Your items will be marked wrong if your solution does not appear in the space indicated.
4. Work carefully and do not spend too much time on any one item. All items should be attempted.

I1.1 Find the sum for the following and write it as a basic fraction:

A. $\frac{3}{5} + \frac{7}{12} + \frac{9}{20} + \frac{11}{30}$

B. $\begin{array}{r} \frac{5}{7} \\ \frac{3}{4} \\ + \frac{5}{8} \end{array}$

I1.1 - A. (________)

I1.1 - B. (________)

I1.2 Find the difference for the following and write it as a basic fraction.

A. $\frac{6}{7} - \frac{5}{8}$

B. $\begin{array}{r} \frac{7}{9} \\ - \frac{4}{7} \\ \hline \end{array}$

I1.2 - A. (________)

I1.2 - B. (_______)

B2.1 A. Write the following as a mixed numeral:

i) $\frac{105}{12}$

ii) $\frac{67}{7}$

B2.1 - A. i) (_______)

B2.1 - B.ii) (_______)

B. Write the following mixed numerals as fractions:

i) $17\frac{4}{5}$

ii) $18\frac{2}{3}$

B2.1 - B. i) (______)

B2.1 - B. ii) (______)

I2.2 A. Show, using fractions, that the mixed numeral for

$\frac{102}{7}$ is $14\frac{4}{7}$

B. Use fractions to justify that the fraction for $10\frac{11}{13}$ is $\frac{141}{13}$.

B3.1 Find the sums and write each as a mixed numeral with the fraction parts as a basic fraction.

A. $15\frac{7}{8}$
$+7\frac{3}{4}$

B. $5\frac{2}{5} + 5\frac{7}{10} + 5\frac{5}{8}$

B3.1 - A. (______)

B3.1 - B. (______)

I3.2 Find the differences and write the fraction parts as basic fractions.

A. $7\frac{5}{7} - 5\frac{3}{5}$

I3.2 - A. (______)

B. $19\frac{7}{10}$
$-12\frac{5}{7}$

I3.2 - B. (______)

I4.1 Solve the following condition and write the fraction in the solution as a basic fraction. Show the check.

$n - 8\frac{5}{8} = 3\frac{5}{14}$

I4.1 (______)

TOPIC II (OPERATIONS WITH RATIONAL NUMBERS)

PHASE II

BASIC LEVEL

Students who achieved less than half of the objectives in Phase I are given this material. It is prepared to let you work at the level of difficulty which suits you. Using it will help you prepare for the next test.

On the next test, you will be expected to answer only those questions which relate to BASIC objectives that you did not achieve on the test you have just written. If you achieved an INTERMEDIATE objective, you will not have to work on the basic objective with the same number, (For example, if you got the question relating to objective I3.2 correct on the first test, then you have already achieved objective B3.2).

Use your record page to tell you which objectives you have not yet achieved. Use your new flow chart to guide you through Phase II and to keep a record of what you have done. Use the objectives in phase I materials to help you know what you have to learn. Use the following activities and exercises to practice your skills for the objectives you have not achieved.

BASIC LEVEL

OBJECTIVE B1

For each of the objectives in section 1 that you did not achieve do the following:

- Read the objective and the corresponding description.
- Do the appropriate exercises and CHECK EXERCISES on these pages.

B1.1 Addition of rational numbers named by fractions.

$\frac{5}{3} + \frac{1}{2} + \frac{4}{9}$

To do the addition at the left:

(1) Find the least (smallest) number which 3, 2 and 9 divide into; i.e. the least common multiple of 3, 2 and 9. It is 18. This is the least common denominator. (L.C.D.)

(2) For each fraction, find an equivalent fraction with the common denominator of 18.

$= \frac{30}{18} + \frac{9}{18} + \frac{8}{18}$ $\qquad \frac{5}{3} = \frac{5 \times 6}{3 \times 6} = \frac{30}{18};\ \frac{1}{2} = \frac{1 \times 9}{2 \times 9} = \frac{9}{18};\ \frac{4}{9} = \frac{4 \times 2}{9 \times 2} = \frac{8}{18}$

$= \frac{47}{18}$ $\qquad$ (3) Add the numerators.

Find the following sums, and write each as a basic fraction:

1. $\frac{3}{4} + \frac{5}{6}$ (remember to find the least common denominator for each fraction and to form equivalent fractions with this L.C.D.)

2. $\frac{1}{3} + \frac{7}{9} + \frac{5}{6}$ (You must find the L.C.D. for all three fractions).

3. $\frac{3}{8} + \frac{1}{6} + \frac{5}{12}$

4. $\frac{2}{3} + \frac{1}{4} + \frac{5}{9}$

5. $\frac{3}{7} + \frac{1}{2} + \frac{5}{4}$

6. $\frac{3}{5} = \boxed{\frac{\ }{\ }}$
$\frac{1}{2} = \boxed{\frac{\ }{\ }}$
$+\frac{3}{4} = +\boxed{\frac{\ }{\ }}$

7. $\frac{7}{8}$
$\frac{3}{4}$
$+\ \frac{5}{16}$

B1.2 Subtraction of rational numbers named by fractions.

$\frac{5}{6} - \frac{3}{10}$

To do the subtraction at the left:

(1) Find the least (smallest) number which 6 and 10 divide into; i.e. the least common multiple of 6 and 10. It is 30. This is the least common denominator.

(2) For each fraction, find an equivalent fraction with the common denominator of 30.

$= \frac{25}{30} - \frac{9}{30}$ $\qquad \frac{5}{6} = \frac{5 \times 5}{6 \times 5} = \frac{25}{30};\ \frac{3}{10} = \frac{3 \times 3}{10 \times 3} = \frac{9}{30}$

$= \frac{16}{30}$ (3) Subtract the numerators.

$= \frac{8}{15}$ (4) Give the answer as a basic fraction.

Find the differences, and write each as a basic fraction:

1. $\frac{3}{4} - \frac{1}{3}$ (remember to find the least common denominator for each fraction and to form equivalent fractions each with this denominator.)
2. $\frac{4}{5} - \frac{3}{4}$ (Again the L.C.D. must be found before subtracting).
3. $\frac{5}{8} - \frac{5}{12}$
4. $\frac{5}{6} - \frac{5}{8}$
5. $\frac{8}{9} - \frac{5}{6}$
6. $\begin{array}{r} \frac{5}{8} = \frac{\square}{\square} \\ -\frac{1}{2} = \frac{\square}{\square} \\ \hline \end{array}$
7. $\begin{array}{r} \frac{11}{8} \\ -\frac{3}{4} \\ \hline \end{array}$

- Check your answers with those given at the end of the topic.
- Do the required parts in CHECK EXERCISE B1.

CHECK EXERCISE B1

B1.1 Find the sums and write each as a basic fraction:

a) $\frac{3}{4} + \frac{7}{8} + \frac{1}{2}$

b) $\frac{5}{2} + \frac{3}{4} + \frac{4}{5}$

c) $\frac{3}{8} + \frac{3}{4} + \frac{2}{3}$

d) $\begin{array}{r} \frac{1}{2} \\ \frac{3}{8} \\ +\frac{3}{4} \\ \hline \end{array}$

e) $\begin{array}{r} \frac{3}{4} \\ \frac{1}{12} \\ +\frac{11}{6} \\ \hline \end{array}$

B1.2 Find the differences and write each as a basic fraction:

a) $\frac{3}{4} - \frac{2}{3}$

b) $\frac{3}{2} - \frac{5}{8}$

c) $\frac{5}{9} - \frac{1}{2}$

d) $\begin{array}{r} \frac{7}{6} \\ -\frac{3}{4} \\ \hline \end{array}$

e) $\begin{array}{r} \frac{5}{2} \\ -\frac{3}{5} \\ \hline \end{array}$

- Check your answers with those given at the end of the topic.

  You were successful on CHECK EXERCISE

  B1.1 if you had at least 4 of the 5 parts correct.

  B1.2 if you had at least 4 of the 5 parts correct.
- If you are not certain how to add or subtract rational numbers, or if you were unsuccessful on a CHECK EXERCISE, consult your teacher. Then do the appropriate exercises that follow.
- Otherwise, go on to your next unachieved objective.

EXERCISES B1

B1.1 Find the sums and write each as a basic fraction:

a) $\frac{1}{2} + \frac{5}{6} + \frac{7}{12}$

b) $\frac{3}{8} + \frac{5}{12} + \frac{3}{4}$

c) $\frac{9}{16} + \frac{5}{8} + \frac{1}{2}$

d) $\begin{array}{r} \frac{1}{4} \\ \frac{3}{5} \\ +\ \frac{3}{4} \\ \hline \end{array}$

e) $\begin{array}{r} \frac{5}{8} \\ \frac{5}{6} \\ +\ \frac{2}{3} \\ \hline \end{array}$

B1.2 Find the differences and write each as a basic fraction:

a) $\frac{4}{5} - \frac{2}{5}$

b) $\frac{5}{8} - \frac{1}{2}$

c) $\frac{3}{5} - \frac{1}{4}$

d) $\begin{array}{r} \frac{3}{4} \\ -\ \frac{2}{3} \\ \hline \end{array}$

e) $\begin{array}{r} \frac{5}{6} \\ -\ \frac{3}{4} \\ \hline \end{array}$

- Check your answers with those given at the end of the topic.
- Go on to your next unachieved objective.

OBJECTIVE B2

- Read objective B2.1 and its description in section 2.
- Do the following exercises.

1. Write the following fractions as mixed numerals:

   a) $\frac{10}{3}$ (remember: $\frac{10}{3} = \frac{9}{3} + \frac{1}{3} = 3 + \frac{1}{3} =$ ______)

   or $3\overline{)10}$ with quotient 3 i.e. $3\frac{1}{3}$

   b) $\frac{11}{3}$ (remember: $\frac{11}{3} = \frac{9}{3} + \frac{2}{3} =$ ____ $=$ ____)

   or $3\overline{)11}$ with quotient 3 rem. 2 i.e. $3\frac{2}{3}$

   c) $\frac{11}{8}$ f) $\frac{15}{4}$

   d) $\frac{22}{7}$ g) $\frac{33}{10}$

   e) $\frac{5}{2}$

2. Write the following mixed numerals as fractions:

   a) $3\frac{1}{2}$ (remember: $3\frac{1}{2} = 3 + \frac{1}{2} = \frac{6}{2} + \frac{1}{2} =$ ______

   or $3\frac{1}{2} = 3 + \frac{1}{2} =$ 6 halves + 1 half $= \frac{7}{2}$)

   b) $2\frac{4}{5}$ (remember: $2\frac{4}{5} = 2 + \frac{4}{5} = \frac{10}{5} + \frac{\quad}{} =$ ____

   or $2\frac{4}{5} = 2 + \frac{4}{5} =$ 10 fifths + 4 fifths = 14 fifths $= \frac{14}{5}$

   c) $1\frac{2}{3}$ f) $3\frac{3}{7}$

   d) $6\frac{2}{3}$ g) $1\frac{3}{8}$

   e) $7\frac{1}{2}$

- Check your answers with those given at the end of the topic.
- Do CHECK EXERCISE B2.

CHECK EXERCISE B2

B2.1 i) Write the following mixed numerals as fractions:

a) $3\frac{2}{5}$ d) $4\frac{4}{5}$

b) $1\frac{5}{8}$ e) $3\frac{1}{4}$

c) $2\frac{1}{2}$

B2.1 ii) Write the following fractions as mixed numerals:

a) $\frac{11}{4}$ d) $\frac{13}{3}$

b) $\frac{15}{8}$ e) $\frac{17}{8}$

c) $\frac{25}{4}$

- Check your answers with those given at the end of the topic.
- If you are unsure how to do the above exercises or if you had more than one part incorrect in either question (i) or question (ii); consult your teacher. Then do the following exercises.
- Otherwise, go on to your next unachieved objective.

EXERCISES B2

B2.1 i) Write the following mixed numerals as fractions:

a) $3\frac{1}{2}$ d) $4\frac{1}{4}$

b) $5\frac{3}{10}$ e) $1\frac{5}{6}$

c) $1\frac{3}{10}$

ii) Write the following fractions as mixed numerals:

a) $\frac{25}{8}$ d) $\frac{19}{7}$

b) $\frac{11}{6}$ e) $\frac{14}{5}$

c) $\frac{8}{3}$

- Check your answers with those given at the end of the topic.
- Go on to your next unachieved objective.

OBJECTIVE B3

For each of the objectives in section 3 that you did not achieve, do the following:

- Read the objective and corresponding description.
- Do the appropriate exercises and CHECK EXERCISES.

B3.1 Find the sums and write each as a mixed numeral with fractional part as a basic fraction:

a) $\begin{array}{r} 7\frac{1}{4} \\ +\ 2\frac{1}{2} \\ \hline \end{array}$ (Remember: $\begin{array}{rcl} 7\frac{1}{4} & = & 7 + \frac{1}{4} \\ +\ 2\frac{1}{2} & = & 2 + \frac{1}{2} \\ \hline & & 9 + \frac{3}{4} = \underline{\quad\quad} \end{array}$)

b) $7\frac{5}{6} + 4\frac{2}{3}$ (Remember: $7\frac{5}{6} + 4\frac{2}{3} = 7 + \frac{5}{6} + 4 + \frac{4}{6}$
$= 11 + \frac{9}{6}$
$= 11 + \underline{\quad\quad}$
$= \underline{\quad\quad}$)

c) $6\frac{1}{12} + 5\frac{1}{6} + 9\frac{2}{3}$

d) $8\frac{3}{5} + 13\frac{3}{4} + 3\frac{7}{10}$

e) $7\frac{1}{3} + 5\frac{3}{5} + 2\frac{1}{15}$

f) $\begin{array}{r} 4\frac{7}{12} \\ 9\ \frac{1}{2} \\ +\ 17\ \frac{3}{8} \\ \hline \end{array}$

g) $\begin{array}{r} 4\frac{5}{16} \\ 14\ \frac{1}{2} \\ +\ \ 3\ \frac{3}{8} \\ \hline \end{array}$

B3.2 Find the differences and write each as a mixed numeral with fractional part as a basic fraction.

a) $\begin{array}{rcl} 5\frac{3}{4} & = & 5 + \frac{9}{12} \\ -\ 2\frac{2}{3} & = & -2 + \underline{\quad} \\ \hline & & 3 + \underline{\quad} = \underline{\quad} \end{array}$

b) $5\frac{1}{4} - 2\frac{3}{4} = \overset{(5-2)}{3} + \frac{1}{4} - \frac{3}{4}$
$= 2 + 1\frac{1}{4} - \frac{3}{4}$
$= 2 + \underline{\quad} - \frac{3}{4}$
$= 2 + \underline{\quad}$
$= 2\frac{1}{2}$

c) $\begin{array}{r} 6\frac{3}{8} \\ -4\frac{3}{4} \\ \hline \end{array}$ d) $\begin{array}{r} 14\frac{4}{5} \\ -13\frac{1}{2} \\ \hline \end{array}$ e) $19\frac{1}{2} - 12\frac{5}{7}$

f) $9 - 5\frac{2}{3}$

- Check your answers with those given at the end of the topic.
- Do the required parts in CHECK EXERCISE B3.

CHECK EXERCISE B3

B3.1 Find the sums and write each as a mixed numeral with fractional part as a basic fraction:

a) $6\frac{2}{3} + 4\frac{3}{4} + 5\frac{5}{6}$

b) $10\frac{3}{4} + 4\frac{1}{12} + 2\frac{5}{6}$

c) $7\frac{4}{5} + 8\frac{3}{4} + 2\frac{3}{10}$

d) $\begin{array}{r} 17\frac{1}{3} \\ 5\frac{5}{6} \\ +\ 4\frac{1}{2} \\ \hline \end{array}$ e) $\begin{array}{r} 3\ \frac{1}{8} \\ 2\frac{1}{16} \\ +\ 1\ \frac{1}{2} \\ \hline \end{array}$

B3.2 Find the differences and write each as a mixed numeral with fractional part as a basic fraction:

a) $5\frac{1}{2} - 3\frac{3}{8}$

b) $4\frac{3}{4} - 2\frac{1}{6}$

c) $17\frac{3}{4} - 6\frac{7}{8}$

d) $\begin{array}{r} 9\frac{1}{2} \\ -7\frac{5}{8} \\ \hline \end{array}$ e) $\begin{array}{r} 3\ \frac{3}{5} \\ -1\frac{9}{10} \\ \hline \end{array}$

- Check your answers with those given at the end of the topic. You were successful on CHECK EXERCISE
  B3.1 if you had at least 4 of the parts correct,
  B3.2 if you had at least 4 of the parts correct.
- If you are unsure how to add or subtract with mixed numerals, or if you were unsuccessful on either CHECK EXERCISE; consult your teacher. Then do the appropriate exercise that follows.
- Otherwise, go on to your next unachieved objective.

EXERCISES B3

B3.1 Find the sums and write each as a mixed numeral with fractional part as a basic fraction.

a) $3\frac{1}{12} + 5\frac{1}{6} + 7\frac{2}{3}$

b) $8\frac{1}{2} + 7\frac{3}{4} + 12\frac{5}{8}$

c) $7\frac{2}{3} + 11\frac{3}{4} + 8\frac{5}{9}$

d) $$\begin{array}{r} 11\frac{5}{9} \\ 8\frac{1}{3} \\ +\ 7\frac{5}{6} \\ \hline \end{array}$$

e) $$\begin{array}{r} 22\ \frac{2}{9} \\ 6\frac{5}{12} \\ +\ 4\ \frac{1}{4} \\ \hline \end{array}$$

B3.2 Find the differences and write each as a mixed numeral with fractional part as a basic fraction.

a) $6\frac{2}{3} - 2\frac{3}{8}$

b) $8\frac{1}{4} - 3\frac{2}{3}$

c) $15\frac{3}{8} - 9\frac{5}{12}$

d) $$\begin{array}{r} 7\frac{1}{2} \\ -\ 3\frac{2}{5} \\ \hline \end{array}$$

e) $$\begin{array}{r} 5\frac{1}{4} \\ -\ 2\frac{5}{6} \\ \hline \end{array}$$

- Check your answers with those given at the end of the topic.
- Go on to your next unachieved objective.

OBJECTIVE B4

- Read objective B4.1 and the corresponding description.
- Do the following exercises.

B4.1 Solution of conditions for equality in which the universe or replacement set for the variable is the set of rational numbers.

$3\frac{4}{5} + n = 5\frac{1}{3}$ To solve the condition at the left:

$n = 5\frac{1}{3} - 3\frac{4}{5}$ (1) form the corresponding condition involving subtraction.

$= 2 + \frac{5}{15} - \frac{12}{15}$ (2) do the subtraction.

$= 1 + \frac{20}{15} - \frac{12}{15}$

$= 1\frac{8}{15}$

Check: $3\frac{4}{5} + 1\frac{8}{15}$ (3) check the solution by replacing n.

$= 4 + \frac{12}{15} + \frac{8}{15}$

$= 4 + \frac{20}{15}$

$= 5\frac{5}{15}$

$= 5\frac{1}{3}$

Solution: $1\frac{8}{15}$ (4) Give the solution.

Solve the following conditions of equality and write fractions in solutions as basic fractions. The universe or replacement set for each variable is the set of rational numbers.

a) $7\frac{4}{5} + n = 8\frac{3}{4}$

$n = 8\frac{3}{4} - 7\frac{4}{5}$ form the corresponding condition involving subtraction.

$= 1 + \frac{15}{20} + \boxed{\frac{\ }{\ }}$

$= \frac{\boxed{\ }}{20} - \frac{16}{20}$

$n = \boxed{\frac{\ }{\ }}$

Check:

Solution:

b) $n - 6\frac{1}{6} = 2\frac{2}{3}$ (remember: form the corresponding condition involving addition:)

$n = \square \; \square \; \bigcirc$

$= 8 + \square + \bigcirc$

$= \frac{\square}{\square}$

c) $n + 7\frac{1}{8} = 9\frac{1}{2}$

d) $14\frac{7}{8} = n - 9\frac{3}{4}$

e) $14\frac{1}{5} = n + 10\frac{5}{6}$

f) $n - 6\frac{3}{8} = 7\frac{3}{4}$

- Check your answers with those given at the end of the topic.
- Do CHECK EXERCISE B4.

CHECK EXERCISE B4

B4.1 Solve the following conditions of equality and write the fractions in solutions as basic fractions. The replacement set or universe for each variable is the set of rational numbers.

a) $n - 8\frac{3}{4} = 7\frac{4}{5}$

b) $6\frac{1}{6} + n = 7\frac{2}{3}$

c) $n - 4\frac{3}{4} = 2\frac{1}{3}$

d) $8\frac{4}{5} = n + 6\frac{2}{3}$

e) $7\frac{5}{7} = n + 2\frac{3}{14}$

- Check your answers with those given at the end of the topic.
- If you are not sure how to solve these conditions, or if you had more than one error; consult your teacher, then do the following exercises.
- Otherwise, go on to your next unachieved objective.

EXERCISES B4

B4.1 Solve the following conditions of equality and write the fractions in solutions as basic fractions. The replacement set or universe for each variable is the set of rational numbers.

a) $n - 2 = 5\frac{6}{7}$

b) $16\frac{1}{2} = n - 12\frac{3}{4}$

c) $9\frac{1}{4} = n + 6\frac{3}{5}$

d) $4\frac{3}{8} + n = 5\frac{3}{4}$

e) $n - 5\frac{1}{2} = 8\frac{7}{10}$

- Check your answers with those given at the end of the topic.
- Go on to your next unachieved objective.

Answers

OBJECTIVE B1

B1.1 1. $\frac{19}{12}$ 2. $\frac{35}{18}$ 3. $\frac{23}{24}$ 4. $\frac{53}{36}$ 5. $\frac{61}{28}$ 6. $\frac{12}{20}, \frac{10}{20}, \frac{15}{20}; \frac{37}{20}$
7. $\frac{31}{16}$

B1.2 1. $\frac{5}{12}$ 2. $\frac{1}{20}$ 3. $\frac{5}{24}$ 4. $\frac{5}{24}$ 5. $\frac{1}{18}$ 6. $\frac{5}{8}, \frac{4}{8}; \frac{1}{8}$ 7. $\frac{5}{8}$

CHECK EXERCISE B1

B1.1 a) $\frac{17}{8}$ b) $\frac{81}{20}$ c) $\frac{43}{24}$ d) $\frac{13}{8}$ e) $\frac{8}{3}$

B1.2 a) $\frac{1}{12}$ b) $\frac{7}{8}$ c) $\frac{1}{18}$ d) $\frac{5}{12}$ e) $\frac{19}{10}$

EXERCISES B.1

B1.1 a) $\frac{23}{12}$ b) $\frac{37}{24}$ c) $\frac{27}{16}$ d) $\frac{8}{5}$ e) $\frac{51}{24}$

B1.2 a) $\frac{2}{5}$ b) $\frac{1}{8}$ c) $\frac{7}{20}$ d) $\frac{1}{12}$ e) $\frac{1}{12}$

OBJECTIVE B2

B2.1 1. a) $3\frac{1}{3}$ b) $3\frac{2}{3}$ c) $1\frac{3}{8}$ d) $3\frac{1}{7}$ e) $2\frac{1}{2}$ f) $3\frac{3}{4}$ g) $3\frac{3}{10}$
2. a) $\frac{7}{2}$ b) $\frac{14}{5}$ c) $\frac{5}{3}$ d) $\frac{20}{3}$ e) $\frac{15}{2}$ f) $\frac{24}{7}$ g) $\frac{11}{8}$

CHECK EXERCISE B2

B2.1 i) a) $\frac{17}{5}$ b) $\frac{13}{8}$ c) $\frac{5}{2}$ d) $\frac{24}{5}$ e) $\frac{13}{4}$
ii) a) $2\frac{3}{4}$ b) $1\frac{7}{8}$ c) $6\frac{1}{4}$ d) $4\frac{1}{3}$ e) $2\frac{1}{8}$

EXERCISES B2

B2.1 i) a) $\frac{7}{2}$ b) $\frac{53}{10}$ c) $\frac{13}{10}$ d) $\frac{17}{4}$ e) $\frac{11}{6}$
ii) a) $3\frac{1}{8}$ b) $1\frac{5}{6}$ c) $2\frac{2}{3}$ d) $2\frac{5}{7}$ e) $2\frac{4}{5}$

OBJECTIVE B3

B3.1 a) $9\frac{3}{4}$ b) $12\frac{1}{2}$ c) $20\frac{11}{12}$ d) $26\frac{1}{20}$ e) 15 f) $31\frac{11}{24}$ g) $22\frac{3}{16}$

B3.2 a) $\frac{8}{12}, \frac{1}{12}, 3\frac{1}{12}$ b) $\frac{5}{4}, \frac{2}{4}$ c) $1\frac{5}{8}$ d) $1\frac{3}{10}$ e) $6\frac{11}{14}$ f) $3\frac{1}{3}$

CHECK EXERCISE B3

B3.1 a) $17\frac{1}{4}$ b) $17\frac{2}{3}$ c) $18\frac{17}{20}$ d) $27\frac{2}{3}$ e) $6\frac{11}{16}$

B3.2 a) $2\frac{1}{8}$ b) $2\frac{7}{12}$ c) $10\frac{7}{8}$ d) $1\frac{7}{8}$ e) $1\frac{7}{10}$

2B28

EXERCISES B3

B3.1 a) $15\frac{11}{12}$ b) $28\frac{7}{8}$ c) $27\frac{35}{36}$ d) $27\frac{13}{18}$ e) $32\frac{8}{9}$

B3.2 a) $4\frac{7}{24}$ b) $4\frac{7}{12}$ c) $5\frac{23}{24}$ d) $4\frac{1}{10}$ e) $2\frac{5}{12}$

OBJECTIVE B4

B4.1 a) $\frac{16}{20}$, 35; $n = \frac{19}{20}$ b) $2\frac{2}{3} + 6\frac{1}{6}$, $\frac{4}{6} + \frac{1}{6}$; $n = 8\frac{5}{6}$ c) $n = 2\frac{3}{8}$

d) $n = 24\frac{5}{8}$ e) $n = 3\frac{11}{30}$ f) $n = 14\frac{1}{8}$

CHECK EXERCISE B4

B4.1 a) $n = 16\frac{11}{20}$ b) $n = 1\frac{1}{2}$ c) $n = 7\frac{1}{12}$ d) $n = 2\frac{2}{15}$ e) $n = 5\frac{1}{2}$

EXERCISES B4

B4.1 a) $n = 7\frac{6}{7}$ b) $n = 29\frac{1}{4}$ c) $n = 2\frac{13}{20}$ d) $n = 1\frac{3}{8}$ e) $n = 14\frac{1}{5}$

OBJECTIVE B5

B5.1 a) $n = 3\frac{5}{16} + 5\frac{13}{16} + \frac{1}{16}$, $\frac{5}{16} + \frac{13}{16} + \frac{1}{16}$, $\frac{19}{16}$, $9\frac{3}{16}$; $9\frac{3}{16}$ inches.

b) $497\frac{29}{72}$ lbs. c) $4\frac{7}{10}$ gal. d) $6\frac{5}{8}$ hr. e) $\frac{3}{8}$ inch f) $2\frac{7}{8}$ oz.

CHECK EXERCISE B5

B5.1 a) $14\frac{3}{8}$ lb. b) $6\frac{1}{8}$ yd. c) $10\frac{3}{8}$ in. d) $9\frac{1}{4}$ in. e) $\frac{1}{2}$ mi.

EXERCISES B5

B5.1 a) $\frac{5}{8}$ mi. b) $3\frac{1}{8}$ lb. c) $\frac{3}{8}$ lb. d) $1\frac{11}{12}$ lb. e) $3\frac{1}{10}$

OBJECTIVE B7

B7.1 a) $\frac{1}{5}$ b) $\frac{2 \times 9}{3 \times 1}$; 6 c) $\frac{13 \times 3}{9 \times 13}$; $\frac{1}{3}$ d) $\frac{3 \times 16}{4 \times 9}$; $1\frac{1}{3}$

e) 6 f) 33 g) $19\frac{4}{5}$ h) $\frac{1}{8}$ i) $9\frac{1}{6}$ j) 21

CHECK EXERCISE B7

B7.1 a) $\frac{7}{12}$ b) $7\frac{1}{2}$ c) 3 d) 45 e) 36

TOPIC II (OPERATIONS WITH RATIONAL NUMBERS)

PHASE II

INTERMEDIATE LEVEL

Students who achieved between half and nine tenths (90%) of the objectives in Phase I are given these materials.

Your job now is to master all of the objectives that you have missed.

On the next test, you will be expected to answer only those questions which are related to objectives that you did not achieve on the first test.

Use your record page to tell you which objectives you need to work on.

Use your flow chart to guide you through Phase II and to show what you have done in Phase I. You may want to do some of the parts you left out during Phase I.

Use your phase I materials to relearn the ideas for objectives you have not achieved and to give you examples of the type of questions you need to be able to answer.

Use these exercises to practice your skills so that you will be able to achieve all of the objectives.

INTERMEDIATE LEVEL

For each of the objectives you did not achieve on Post Test I, do the appropriate exercises and check your answers with those given at the end of the topic. Review the appropriate material in Phase I before starting each exercise.

OBJECTIVE I1.1

Find the following sums and write each as a basic fraction:

a) $\frac{5}{6} + \frac{3}{4} + \frac{4}{5}$

b) $\frac{3}{7} + \frac{1}{2} + \frac{2}{3}$

c) $\frac{5}{8} + \frac{5}{6} + \frac{5}{9}$

d) $\frac{4}{9} + \frac{5}{12} + \frac{2}{15}$

e) $\frac{4}{15} + \frac{3}{20} + \frac{5}{6}$

OBJECTIVE I1.2

Find the differences and write each as a basic fraction:

a) $\frac{8}{9} - \frac{3}{5}$

b) $\frac{4}{11} - \frac{1}{7}$

c) $\frac{9}{16} - \frac{3}{20}$

d) $\frac{14}{25} - \frac{4}{15}$

e) $\frac{17}{24} - \frac{3}{16}$

OBJECTIVE B2.1

i) Write the following mixed numerals as fractions:

a) $3\frac{1}{2}$

b) $5\frac{3}{10}$

c) $1\frac{3}{8}$

d) $4\frac{1}{4}$

e) $7\frac{5}{6}$

ii) Write the following fractions as mixed numerals:

a) $\frac{25}{8}$

b) $\frac{11}{6}$

c) $\frac{28}{3}$

d) $\frac{19}{7}$

e) $\frac{34}{5}$

OBJECTIVE I5.1

Solve the following applied problems:

a) Carol worked as a babysitter one summer. During one week she worked $3\frac{1}{2}$ hours on Monday, $4\frac{2}{3}$ hours on Tuesday, $2\frac{4}{5}$ hours on Wednesday, $5\frac{1}{6}$ hours on Saturday and $3\frac{3}{10}$ hours on Sunday. How many hours did Carol work during the week?

b) Four sections of a highway totaling $15\frac{2}{3}$ miles are to be built. Three of the sections measure $3\frac{1}{2}$, $5\frac{3}{5}$ and $5\frac{1}{6}$ miles. What is the length of the fourth section?

c) John, who weighed $135\frac{3}{4}$ pounds, went on a diet. After the first week he lost $2\frac{2}{3}$ pounds. However, during the second week he gained $1\frac{2}{5}$ pounds. How much did he weigh at the end of the second week?

d) A group of scouts joined short lengths of rope together in order to descend a steep bank. The lengths measured $3\frac{1}{2}$ ft., $2\frac{3}{3}$ ft. 18 inches, and $4\frac{3}{8}$ ft. How many feet long was the joined rope? (Allow 1 foot for the knots.)

e) George and Bill leave their respective homes, $3\frac{2}{5}$ miles apart, planning to meet half-way. George walked $\frac{3}{4}$ of a mile then stopped to chat with a friend. He was still talking when Bill stopped for a bottle of pop $1\frac{1}{8}$ miles from his home. How far apart were the boys when they both stopped?

OBJECTIVE I6.1

a) Show by diagrams how $\frac{2}{3}$ of $\frac{3}{4}$ can be obtained.

b) Show by diagrams how $\frac{4}{5}$ of $\frac{2}{3}$ can be obtained.

OBJECTIVE I7.1

Find the following products:

a) $\frac{8}{9}$ of $7\frac{7}{8}$

b) $\frac{7}{8}$ of $\frac{4}{21}$

c) $\frac{14}{19} \times 3\frac{1}{7} \times 2\frac{3}{8}$

d) $\frac{4}{15} \times 22\frac{1}{2} \times 2\frac{2}{3}$

e) $2\frac{1}{4} \times 3\frac{1}{7} \times \frac{4}{27} \times 3$

OBJECTIVE I2.2

i) a) Use fractions to justify that the mixed numeral for $\frac{33}{7}$ is $4\frac{5}{7}$.

b) Use fractions to justify that the mixed numeral for $\frac{25}{3}$ is $8\frac{1}{3}$.

ii) a) Use fractions to justify that the fraction for $5\frac{2}{7}$ is $\frac{37}{7}$.

b) Use fractions to justify that the fraction for $6\frac{3}{7}$ is $\frac{45}{7}$.

OBJECTIVE I3.1

Find the following sums and write each as a mixed numeral with fractional part a basic fraction:

a) $9\frac{2}{3} + 13\frac{3}{4} + 7\frac{4}{5}$

b) $12\frac{7}{10} + 19\frac{3}{4} + 7\frac{7}{12}$

c) $4\frac{5}{16} + 17\frac{5}{9} + 10\frac{5}{12}$

d) $\begin{array}{r} 6\frac{7}{10} \\ 12\frac{4}{15} \\ +\ 2\frac{5}{6} \\ \hline \end{array}$

e) $\begin{array}{r} 11\frac{7}{9} \\ \frac{5}{6} \\ +\ 4\frac{3}{4} \\ \hline \end{array}$

OBJECTIVE I3.2

Find the differences and write the fraction parts of the mixed numerals as basic fractions:

a) $5\frac{7}{12} - 3\frac{8}{9}$

b) $11\frac{7}{9} - 3\frac{5}{6}$

c) $7 - 4\frac{2}{5}$

d) $\begin{array}{r} 8\frac{5}{24} \\ -\ 3\frac{7}{16} \\ \hline \end{array}$

e) $\begin{array}{r} 12\frac{11}{18} \\ -\ \frac{11}{12} \\ \hline \end{array}$

OBJECTIVE I4.1

Solve the following conditions of equality. The universe or replacement set for each variable is the set of rational numbers.

a) $3\frac{7}{9} = n + 2\frac{3}{4}$

b) $1\frac{1}{6} + p = 4\frac{1}{8}$

c) $m - 4\frac{5}{16} = 17\frac{5}{9}$

d) $12\frac{7}{15} = a - 9\frac{8}{9}$

e) $6\frac{3}{4} = 7\frac{2}{3} - z$

Answers

OBJECTIVE I1.1

a) $\frac{143}{60}$ b) $\frac{67}{42}$ c) $\frac{145}{72}$ d) $\frac{179}{180}$ e) $\frac{5}{4}$

OBJECTIVE I1.2

a) $\frac{13}{45}$ b) $\frac{17}{77}$ c) $\frac{33}{80}$ d) $\frac{22}{75}$ e) $\frac{25}{48}$

OBJECTIVE B2.1

i) a) $\frac{7}{2}$ b) $\frac{53}{10}$ c) $\frac{11}{8}$ d) $\frac{17}{4}$ e) $\frac{47}{6}$

ii) a) $3\frac{1}{8}$ b) $1\frac{5}{6}$ c) $9\frac{1}{3}$ d) $2\frac{5}{7}$ e) $6\frac{4}{5}$

OBJECTIVE I2.2

i) a) $\frac{33}{7} = \frac{28}{7} + \frac{5}{7} = \frac{4}{1} + \frac{5}{7} = 4 + \frac{5}{7} = 4\frac{5}{7}$

b) $\frac{25}{3} = \frac{24}{3} + \frac{1}{3} = \frac{8}{1} + \frac{1}{3} = 8 + \frac{1}{3} = 8\frac{1}{3}$

ii) a) $5\frac{2}{7} = 5 + \frac{2}{7} = \frac{35}{7} + \frac{2}{7} = \frac{37}{7}$

b) $6\frac{3}{7} = 6 + \frac{3}{7} = \frac{42}{7} + \frac{3}{7} = \frac{45}{7}$

OBJECTIVE I3.1

a) $31\frac{13}{60}$ b) $40\frac{1}{30}$ c) $32\frac{41}{144}$ d) $21\frac{4}{5}$ e) $17\frac{13}{36}$

OBJECTIVE I3.2

a) $1\frac{25}{36}$ b) $7\frac{17}{18}$ c) $2\frac{3}{5}$ d) $4\frac{37}{48}$ e) $11\frac{25}{36}$

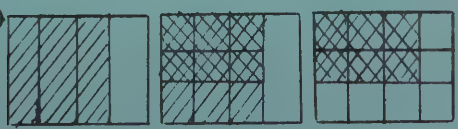
OBJECTIVE I4.1

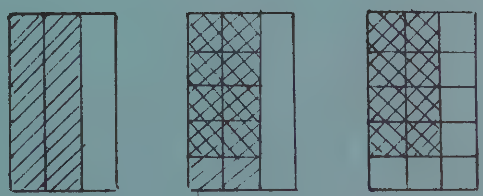
a) $n = 1\frac{1}{36}$ b) $p = 2\frac{23}{24}$ c) $m = 21\frac{125}{144}$ d) $a = 22\frac{16}{45}$ e) $z = \frac{11}{12}$

OBJECTIVE I5.1

a) $19\frac{13}{30}$ hr. b) $1\frac{2}{5}$ mi. c) $134\frac{29}{60}$ lb. d) $11\frac{1}{24}$ ft. e) $1\frac{21}{40}$ miles

OBJECTIVE I6.1

a) 

b) 

OBJECTIVE I7.1

a) 7 b) $\frac{1}{6}$ c) $5\frac{1}{2}$ d) 16 e) $3\frac{1}{7}$

OBJECTIVE I8.1

i) The product of a given number and its reciprocal is one.

ii) a) $\frac{3}{2}$ b) $\frac{1}{4}$ c) 1 d) 2 e) $\frac{2}{7}$

iii) Zero does not have a reciprocal as there is no number which multiplied by 0 gives a product of 1.

OBJECTIVE I9.1

a) $4\frac{2}{3}$ b) 45 c) $3\frac{4}{7}$ d) $\frac{27}{64}$ e) 0 f) 24 g) $\frac{3}{4}$

OBJECTIVE I9.2

$\frac{7}{9} \div \frac{0}{3}$ is not possible as to divide, we replace the division by multiplication by the reciprocal of the divisor, and $\frac{0}{3}$ does NOT have a reciprocal.

OBJECTIVE I10.1

a) $m = 22$ b) $p = 2\frac{1}{4}$ c) $q = 2\frac{2}{3}$ d) $n = 1\frac{11}{14}$ e) $a = 1\frac{35}{36}$

OBJECTIVE I11.1

a) 9 bags b) 260 mi. c) $4\frac{5}{24}$ d) 5 cups

e) $51\frac{9}{20}$ dollars f) $3\frac{3}{5}$

OBJECTIVE I12.1

a) $\frac{1}{2} + (\frac{2}{3} + \frac{1}{2}) = (\frac{1}{2} + \frac{2}{3}) + \frac{1}{2}$

b) $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$

c) $\frac{2}{5} + 0 = \frac{2}{5}$

d) $\frac{3}{4} + \frac{1}{3} = \frac{1}{3} + \frac{3}{4}$

or any similar answers

OBJECTIVE I12.2

i) a) commutative b) associative c) identity d) every non-zero rational number has a reciprocal e) closure f) zero product.

ii) distributive property of multiplication over addition.

OBJECTIVE I12.3

i) every non-zero rational number has a reciprocal.

ii) 5 x ___ = 1 - there is no number to multiply by 5 to get a product of 1.

TOPIC II (OPERATIONS WITH RATIONAL NUMBERS)

PHASE II

ADVANCED LEVEL

Students who achieved nine tenths (90%) or more of the objectives for Phase I are given this material.

First, check your record page. If you missed any of the objectives there, go back through the materials and relearn the proper sections. You will be expected to answer the questions that relate to those objectives on the next test.

In this packet, there are several new objectives that you will be asked to achieve. Since there are only a few of these objectives, they have been collected at the beginning of the section. You should find them more interesting and challenging than the phase I objectives. You will be expected to answer questions on these objectives on the next test.

2A1

OBJECTIVE A1

To state the definition of a non-negative rational number.

Criterion: Statement to include the same ideas as:
"A non-negative rational number is a number named by a fraction of the form $\frac{a}{b}$ where a is a whole number and b is a non-zero whole number."

OBJECTIVE A2

To write the definitions for addition and subtraction of rational numbers named by fractions.

Criterion: Statements including the same ideas as:
"For any rational numbers $\frac{a}{b}$ and $\frac{c}{d}$,

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd} \text{ ,"}$$

OBJECTIVE A3

To write the definition for multiplication of rational numbers named by fractions.

Criterion: Statement including the same ideas as:
For any rational numbers $\frac{a}{b}$ and $\frac{c}{d}$,

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \text{ ."}$$

OBJECTIVE A4

To write the definition for division of rational numbers named by fractions.

Criterion: Statement including the same ideas as:
"For any rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ $(\frac{c}{d} \neq 0)$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

OBJECTIVE A5

To write the statements or reasons which complete the proof of a given property of operations for rational numbers.

Example:

For each of the numbered spaces in the proof below write the statement or reason which is missing.

Prove that addition of rational numbers is commutative:

Let $\frac{a}{b}$ and $\frac{c}{d}$ be any rational numbers.

Prove: $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$

Proof:

Statement	Reason
$\frac{a}{b} + \frac{c}{d}$	
$= \_\_\_\_\_$ (1)	Definition of addition of rational
$= \frac{da + cb}{db}$	numbers __________ (2)
$= \frac{\_\_\_ + \_\_\_}{db}$ (3)	Addition of whole numbers is commutative
$= \frac{c}{d} + \frac{a}{b}$	__________ (4)
$\therefore\ \frac{a}{b} + \frac{c}{d} = \frac{c}{a} + \frac{a}{b}$	

i.e. Addition of rational numbers is commutative.

Criterion: 75% of computations correct.

SOLUTION

(1)	$\frac{ad + bc}{bd}$
(2)	Multiplication of rational numbers is commutative.
(3)	cb + da
(4)	Definition of addition of rational numbers.

2A3

OBJECTIVE A6

To use the distributive property to obtain products of rational numbers.

Example

a) Show how the distributive property may be used to simplify

$$(\frac{2}{3} \times \frac{2}{5}) + (\frac{7}{3} \times \frac{2}{5})$$

b) Show how the distributive property may be used to find the product

$$\frac{1}{2} \times 8\frac{2}{3}$$

c) Find the following product without changing the mixed numerals to fractions:

$$\begin{array}{r} 3\frac{1}{4} \\ \times\ 8\frac{2}{3} \\ \hline \end{array}$$

Criterion: Method correct in each example and no more than one error in computation.

SOLUTION

a) $(\frac{2}{3} \times \frac{2}{5}) + (\frac{7}{3} \times \frac{2}{5}) = (\frac{2}{3} + \frac{7}{3}) \times \frac{2}{5} = \frac{9}{3} \times \frac{2}{5} = 3 \times \frac{2}{5} = \frac{6}{5}$

b) $\frac{1}{2} \times 8\frac{2}{3} = \frac{1}{2} \times (8 + \frac{2}{3}) = (\frac{1}{2} \times 8) + (\frac{1}{2} \times \frac{2}{3}) = 4 + \frac{1}{3} = 4\frac{1}{3}$

c)

$$\begin{array}{r} 3\frac{1}{6} \\ \times\ 4\frac{2}{3} \\ \hline 2\frac{2}{18} \\ 12\ \frac{4}{6} \\ \hline 14\frac{14}{18} \end{array} = 14\frac{7}{9}$$

1. DEFINITION FOR RATIONAL NUMBERS

In mathematics, a definition is a statement which tells precisely what something means.

In this section we will see the definition for rational numbers.

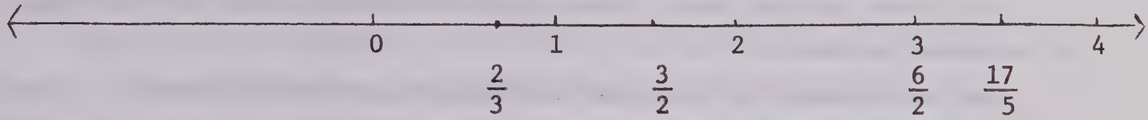
You have seen that each rational number is associated with an infinite set of equivalent fractions each of which has the form $\frac{a}{b}$ where a is a whole number and b is a non-zero whole number (or natural number).

e.g. The rational number two thirds is associated with the infinite set of fractions $\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\}$

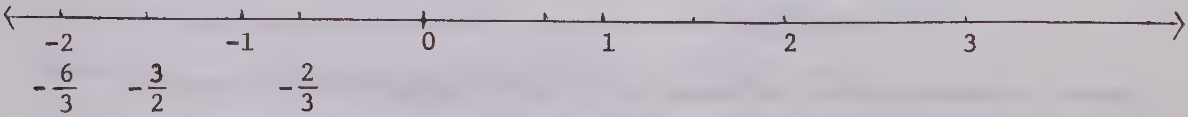
This gives us a way of defining rational numbers as follows:

<u>A rational number is a number named by a fraction of the form $\frac{a}{b}$ where a is a whole number and b is a non-zero whole number.</u>

This defines the sort of rational number which we have been using. That is, 0 and the rational numbers associated with points to the right of the point associated with 0 on the number line.



There are however rational numbers associated with points to the left of the point associated with 0 on the number line. Some of these are shown below.



Note the symbols used for these new rational numbers.

The two types of rational numbers are distinguished by calling those associated with points to the right of the point associated with 0 the POSITIVE RATIONAL NUMBERS and those associated with points to the left of the point associated with 0 the NEGATIVE RATIONAL NUMBERS.

You will learn more about the negative rational numbers later.

0 is neither a positive rational number nor a negative rational number.

0 and the positive rational numbers make up a set called the set of non-negative rational numbers (i.e. the rational numbers which are not negative).

The set of rational numbers we defined earlier is actually this set of non-negative rational numbers.

2. DEFINITIONS FOR ADDITION AND SUBTRACTION OF RATIONAL NUMBERS

In this section we will give definitions of addition and subtraction of rational numbers named by fractions; i.e. mathematical statements which give precisely the meaning of addition and subtraction of rational numbers named by fractions.

Apart from stating precisely what something means, a definition is a general statement which represents all possible particular instances of the thing being defined.

Let's see how the above ideas relate to addition and subtraction of rational numbers.

The definitions for addition and subtraction of rational numbers named by fractions are:

"For any rational numbers $\frac{a}{b}$ and $\frac{c}{d}$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$ "

Note: ad means a x d, bc means b x c and bd means b x d. Pairs of letters written together as in the definitions indicate multiplication.

These may not look quite like the addition or subtraction we did in SECTION 1, however, they are general statements and do give the result for any addition or subtraction example.

1. Check that the sum of $\frac{5}{6}$ and $\frac{2}{5}$ is the same by the definition and by the common denominator method.
2. Check that the difference of $\frac{5}{6}$ and $\frac{3}{4}$ is the same by the definition and by the common denominator method.

Using the definition, you may not get the result as a basic fraction. However, a fraction and its equivalent basic fraction do name the same rational number.

Either the definition, or the common denominator method can be used to find the sum or difference of two rational numbers. We usually use the common denominator method (least common denominator in fact). However this method cannot be readily stated in the form of a general definition.

3. DEFINITION OF MULTIPLICATION OF RATIONAL NUMBERS

The definition for multiplication of rational numbers named by fractions is as given in OBJECTIVE A7.1.

As with the definitions for addition and subtraction of rational numbers named by fractions, the product may not be a basic fraction. However, we saw in SECTION 7 how the product can be reduced to a basic fraction.

Answers:

1. $\frac{5}{6} + \frac{2}{5} = \frac{5 \times 5 + 6 \times 2}{6 \times 5} = \frac{25 + 12}{30} = \frac{37}{30}$

   $\frac{5}{6} + \frac{2}{5} = \frac{25}{30} + \frac{12}{30} = \frac{37}{30}$

2. $\frac{5}{6} - \frac{3}{4} = \frac{5 \times 4 - 6 \times 3}{6 \times 4} = \frac{20 - 18}{24} = \frac{2}{24} = \frac{1}{12}$

   $\frac{5}{6} - \frac{3}{4} = \frac{10}{12} - \frac{9}{12} = \frac{1}{12}$

4. DEFINITION OF DIVISION OF RATIONAL NUMBERS

The definition for division of rational numbers named by fractions is as given in OBJECTIVE A9.1. Note the condition that the divisor cannot be zero since we cannot divide by 0.

5. PROVING PROPERTIES OF OPERATIONS WITH RATIONAL NUMBERS

In SECTION 12, we used examples to check that various properties of addition and multiplication of rational numbers named by fractions held.

We can in fact prove that these properties hold by using

(1) our knowledge of the properties of addition and multiplication of whole numbers (stated in SECTION 12)

and (2) the definitions for addition and multiplication of rational numbers named by fractions (stated in OBJECTIVES A1.1 and A7.1).

Let's see how two of the properties can be proved. Then you can complete the proofs of the others.

I Prove that addition of rational numbers is associative.

Let $\frac{a}{b}$, $\frac{c}{d}$, $\frac{e}{f}$ be any rational numbers.

Prove: $(\frac{a}{b} + \frac{c}{d}) + \frac{e}{f} = \frac{a}{b} + (\frac{c}{d} + \frac{e}{f})$

Proof:

Statement	Reason	Comment
$(\frac{a}{b} + \frac{c}{d}) + \frac{e}{f} = \frac{(ad + bc)}{bd} + \frac{e}{f}$	definition of addition	
$= \frac{(ad + bc)f + (bd)e}{(bd)f}$	definition of addition	(bd)e means (b x d)xe
$= \frac{f(ad + bc) + (bd)e}{(bd)f}$	Mult. of whole numbers is commutative	Change of order.
$= \frac{\{f(ad) + f(bc)\} + (bd)e}{(bd)f}$	distributive property for whole numbers	
$= \frac{\{(ad)f + (bc)f\} + (bd)e}{(bd)f}$	Mult. of whole numbers is commutative	Change of order.
$= \frac{(ad)f + (bc)f + (bd)e}{(bd)f}$	Add. of whole numbers is associative	Grouping for addition does not matter.
$= \frac{adf + bcf + bde}{bdf}$	Mult. of whole numbers is associative.	Grouping for multiplication does not matter.

Statement	Reason	
$\frac{a}{b} + (\frac{c}{d} + \frac{e}{f})$		
$= \frac{a}{b} + \frac{(cf + de)}{df}$	definition of addition	
$= \frac{a(df) + b(cf + de)}{b(df)}$	definition of addition	
$= \frac{a(df) + \{b(cf) + b(de)\}}{b(df)}$	distributive property for whole numbers.	
$= \frac{a(df) + b(cf) + bde}{b(df)}$	Add. of whole numbers is associative.	
$= \frac{adf + bcf + bde}{bdf}$	Mult. of whole numbers is associative.	
$\therefore (\frac{a}{b} + \frac{c}{d}) + \frac{e}{f} = \frac{a}{b} + (\frac{c}{d} + \frac{e}{f})$		The last line in each set of equalities is the same.

$\therefore$ Addition of rational numbers is associative.

$\therefore$ is an abbreviation for 'therefore'. However, it is <u>never</u> used in an English sentence.

A proof, as you see above, is a sequence of statements for each of which a reason is given. Proofs can often be conveniently given in two column form. The first column gives the statements; the second column gives the reasons for the statement.

In our proof above, the reasons used are the definition of addition for rational numbers and the properties of operations for whole numbers.

II Prove that 1 is the identity for multiplication of rational numbers.

Let $\frac{a}{b}$ be any rational number.

Prove: $\frac{a}{b} \times 1 = \frac{a}{b}$

Proof:

Statement	Reason
$\frac{a}{b} \times 1$	$\frac{1}{1}$ is another name for 1.
$= \frac{a}{b} \times \frac{1}{1}$	definition of multiplication
$= \frac{a \times 1}{b \times 1}$	
$= \frac{a}{b}$	1 is the identity for multiplication of whole numbers.

$\therefore$ 1 is the identity for multiplication of rational numbers.

Now turn to and do the activities on the following pages.

SECTION 5 - ACTIVITY

Each of the following is a proof for one of the properties of operations with rational numbers. In these proofs, certain statements or reasons have been omitted and left for you to give. Each place where something has been omitted is marked with an * and numbered. Write your answers in your workbook, NOT on these pages. At the end of each proof, check the answers you gave with the answers given at the end of the advanced work. If you cannot see why a particular answer is given, ask another person in the A group or ask your teacher.

The symbol "$\therefore$" means "therefore".

1. Prove that the rational numbers are closed for addition.

Let $\frac{a}{b}$ and $\frac{c}{d}$ be any rational numbers.

Prove: $\frac{a}{b} + \frac{c}{d}$ is a rational number.

Proof:

Statement	Reason	Comment
$\frac{a}{b} + \frac{c}{d}$		
$= \frac{ad + bc}{bd}$	Definition of addition of rational numbers	
ad and bc are whole numbers	Whole numbers are closed for multiplication	Here we make use of properties of whole numbers.
(ad + bc) is a whole number	______ *1	
bd is a non-zero whole number	$b \neq 0$, $d \neq 0$ and ______ *2	
$\therefore \frac{ad + bc}{bd}$ is a rational number	Definition of rational number.	
i.e. $\frac{a}{b} + \frac{c}{d}$ is a rational number		This comes from the first two lines in the proof.
i.e. The rational numbers are closed for addition.		

2. Prove that the rational numbers are closed for multiplication.

Let $\frac{a}{b}$ and $\frac{c}{d}$ be any rational numbers.

Prove: $\frac{a}{b} \times \frac{c}{d}$ is a rational number.

Proof:

Statement	Reason	Comment
$\frac{a}{b} \times \frac{c}{d}$		
$= \frac{ac}{bd}$	________________ *3	ac means a x c
ac is a whole number	________________ *4	
________________________ *5	$b \neq 0$, $d \neq 0$, and whole numbers are closed for multiplication. ________________ *6	The denominator must be non-zero.
$\therefore \frac{ac}{bd}$ is a rational number.		
i.e. $\frac{a}{b} \times \frac{c}{d}$ is a rational number.		This comes from the first two lines of the proof.
i.e. The rational numbers are closed for multiplication.		

3. Prove that multiplication of rational numbers is commutative.

Let $\frac{a}{b}$ and $\frac{c}{d}$ be any rational numbers.

Prove: $\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$

Proof:

Statement	Reason	
$\frac{a}{b} \times \frac{c}{d}$		
$= \frac{ac}{bd}$	Definition of multiplication of rational numbers	
$= \frac{ca}{db}$	________________ *7	The order is changed.
$=$ ________________ *8	Definition of multiplication of rational numbers.	Here the definition of multiplication of rational numbers is used in the reverse direction to its use in the first two lines of the proof.
$\therefore \frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$		
i.e. ________________ *9		

4. Prove that addition of rational numbers is commutative.

Let $\frac{a}{b}$ and $\frac{c}{d}$ be any rational numbers.

Prove: $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$

Proof:

Statement	Reason	Comment
$\frac{a}{b} + \frac{c}{d}$		
= ______________ *10	Definition of addition of rational numbers.	
$= \frac{da + cb}{db}$	______________ *11	This property is used three times here.
= ______________ *12	Addition of whole numbers is cummutative.	
$= \frac{c}{d} + \frac{a}{b}$	______________ *13	
∴ __________ = ______ *14		
i.e. Addition of rational numbers is commutative		

5. Prove that multiplication of rational numbers is associative.

Let $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ be any rational numbers.

Prove: $(\frac{a}{b} \times \frac{c}{d}) \times \frac{e}{f} = \frac{a}{b} \times (\frac{c}{d} \times \frac{e}{f})$

Proof:

Statement	Reason	Comment
$(\frac{a}{b} \times \frac{c}{d}) \times \frac{e}{f}$		
$= (\frac{ac}{bd}) \times \frac{e}{f}$	______________ *15	The definition is used twice here.
$= \frac{(ac)\ e}{(bd)\ f}$	Definition of multiplication of rational numbers.	
$= \frac{(\quad)}{(\quad)}$ *16	Mult. of whole numbers is associative.	The grouping is changed.
$= \frac{a}{b} \times (\frac{cd}{df})$	______________ *17	
= ______________ *18	Definition of mult. of rational numbers	
$\therefore (\frac{a}{b} \times \frac{c}{d}) \times \frac{e}{f} = \frac{a}{b} \times (\frac{c}{d} \times \frac{e}{f})$		
i.e. ______________ *19		

Note: In proof I on page of the description we proved that addition of rational numbers is associative.

6. Prove that 0 is the identity for addition of rational numbers.

Let $\frac{a}{b}$ be any rational number.

Prove: $\frac{a}{b} + 0 = \frac{a}{b}$

Proof:

Statement	Reason	Comment
$\frac{a}{b} + 0$		
$= \frac{a}{b} + \frac{0}{1}$		$\frac{0}{1}$ is another name for 0.
$= \frac{a \times 1 + b \times 0}{b \times 1}$	______________ *20	
$= \frac{\quad + \quad}{\qquad}$ *21	1 is the identity for mult. of whole numbers and $b \times 0 = 0$	
$= \frac{a}{b}$	______________ *22	Another property of whole numbers.
$\therefore$ ______________ *23		
i.e. 0 is the identity for addition of rational numbers.		

7. Prove that multiplication is distributive over addition of rational numbers.

Let $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f}$ be any rational numbers.

Prove: $\frac{a}{b} \times (\frac{c}{d} + \frac{e}{f}) = (\frac{a}{b} \times \frac{c}{d}) + (\frac{a}{b} \times \frac{e}{f})$

Proof:

Statement	Reason	Comment
$\frac{a}{b} \times (\frac{c}{d} + \frac{e}{f})$		
$= \frac{a}{b} \times (\text{———})$ *24	Definition of addition of rational numbers.	
$= \frac{a\ (cf + de)}{b(df)}$	______________ *25	
$= \frac{a(cf) + a(de)}{b(df)}$	______________ *26	
$= \frac{acf + ade}{bdf}$	Multiplication of whole numbers is associative.	This permits the parentheses to be omitted.

Statement	Reason	Comment
$(\frac{a}{b} \times \frac{c}{d}) + (\frac{a}{b} \times \frac{e}{f})$		Now we start with the right side of what we have to prove and get it into the same form as that to which we changed the left side.
$= \frac{ac}{bd} + \frac{ae}{bf}$	____________ *27	
$= \frac{(ac)(bf) + (bd)(ae)}{(bd)(bf)}$	____________ *28	
$= \frac{acbf + bdae}{bdbf}$	____________ *29	
$= \frac{bacf + bdae}{bdbf}$	Mult. of whole numbers is commutative.	The order in the products in the numerator has been changed.
$= \frac{b(acf + dae)}{bdbf}$	Distributive property for whole numbers.	
$= \frac{\cancel{b}(acf + dae)}{\cancel{b}dbf}$	Numerator and denominator are divided by b.	The fraction is reduced.
$= \frac{acf + dae}{dbf}$		
$= \frac{acf + ade}{bdf}$	____________ *30	The order is changed in two of the products This is now the same as the left side.
$\therefore \frac{a}{b} \times (\frac{c}{d} + \frac{e}{f}) =$ ____________ *31		
i.e. Multiplication is distributive over addition of rational numbers.		

8. Prove that the multiplication property of zero holds for the rational numbers.

Let $\frac{a}{b}$ be any rational number.

Prove: $\frac{a}{b} \times 0 = 0$

Proof:

Statement	Reason	Comment
$\frac{a}{b} \times 0$		
$= \frac{a}{b} \times \frac{0}{1}$		$\frac{0}{1}$ is another name for 0
$=$ ________ *32	Definition of mult. of rational numbers.	
$= \frac{0}{b}$	________ and ________ *33	$\frac{0}{b}$ is another name for 0.
$= 0$		
$\therefore \frac{a}{b} \times 0 = 0$		
i.e. ________________ *34		

6. USE OF THE DISTRIBUTIVE PROPERTY

Multiplication and addition of whole numbers and of rational numbers are related by the distributive property.

We have seen the distributive property in the form:

$a \times (b + c) = (a \times b) + (a \times c)$

Now look at the following:

$(b + c) \times a = a \times (b + c)$ Commutative property of multiplication.

$= (a \times b) + (a \times c)$ Distributive property

$= (b \times a) + (c \times a)$ Commutative property of multiplication.

i.e. $(b + c) \times a = (b \times a) + (c \times a)$

We see that the distributive property can be used when the single multiplier is on the left or right of the parentheses.

Now let us look at some examples in which the distributive property is used.

Examples

(1) $6 \times 37 = 6 \times (30 + 7) = (6 \times 30) + (6 \times 7) = 180 + 42 = 222$

(2) $6 \times 3\frac{1}{4} = 6 \times (3 + \frac{1}{4}) = (6 \times 3) + (6 \times \frac{1}{4}) = 18 + \frac{6}{4} = 18 + \frac{3}{2} = 19\frac{1}{2}$

(3) $\frac{1}{3} \times 6\frac{3}{5} = \frac{1}{3} \times (6 + \frac{3}{5}) = (\frac{1}{3} \times 6) + (\frac{1}{3} \times \frac{3}{5}) = 2 + \frac{1}{5} = 2\frac{1}{5}$

(4) $2\frac{1}{4} \times 5 = (2 + \frac{1}{4}) \times 5 = (2 \times 5) + (\frac{1}{4} \times 5) = 10 + \frac{5}{4} = 10 + 1\frac{1}{4} = 11\frac{1}{4}$

1. The product $3\frac{1}{3} \times 6\frac{1}{2}$ can be written as
$$\begin{array}{r} 6\frac{1}{2} \\ \times\ 3\frac{1}{3} \\ \hline \end{array}$$

Examine the following to see what has been done to find the product.

$$\begin{array}{r} 6\frac{1}{4} \\ \times\ 3\frac{1}{3} \\ \hline 2\frac{1}{12} \\ 18\frac{3}{4} \\ \hline 20\frac{10}{12} \end{array} = 20\frac{5}{6}$$

2. Use the same method as above to show that

$4\frac{1}{3} \times 2\frac{1}{2} = 10\frac{5}{6}$.

In the examples in items 1 and 2 on the previous page, we have also been using the distrubutive property. The following shows how.

$$3\frac{1}{3} \times 6\frac{1}{4} = (3 + \frac{1}{3}) \times 6\frac{1}{4} = (3 \times 6\frac{1}{4}) + (\frac{1}{3} \times 6\frac{1}{4}) = 3 \times (6 + \frac{1}{4}) + \frac{1}{3} \times (6 + \frac{1}{4})$$

$$= (3 \times 6) + (3 \times \frac{1}{4}) + (\frac{1}{3} \times 6) + (\frac{1}{3} \times \frac{1}{4}) = 18 + \frac{3}{4} + 2 + \frac{1}{12}$$

$$= 20 + \frac{9}{12} + \frac{1}{12} = 20 + \frac{10}{12} = 20\frac{5}{6}$$

Now do the exercises on the following activity page.

Answers: 1. $\frac{1}{3} \times 6\frac{1}{4} = \frac{1}{3} \times (6 + \frac{1}{4}) = (\frac{1}{3} \times 6) + (\frac{1}{3} \times \frac{1}{4}) = 2 + \frac{1}{12} = 2\frac{1}{12}$

$3 \times 6\frac{1}{4} = 3 \times (6 + \frac{1}{4}) = (3 \times 6) + (3 \times \frac{1}{4}) = 18 + \frac{3}{4} = 18\frac{3}{4}$

$2\frac{1}{12} + 18\frac{3}{4} = 20 + \frac{1}{12} + \frac{9}{12} = 20 + \frac{10}{12} = 20\frac{5}{6}$

2.
$$\begin{array}{r} 4\frac{1}{3} \\ \times 2\frac{1}{2} \\ \hline 2\frac{1}{6} \\ 8\frac{2}{3} \\ \hline 10\frac{5}{6} \end{array}$$

SECTION 6 - ACTIVITY

1. Use the distributive property to simplify the following:

(a) $(\frac{3}{4} \times \frac{2}{5}) + (\frac{3}{4} \times \frac{3}{5})$

(b) $(\frac{5}{8} \times \frac{5}{6}) + (\frac{5}{8} \times \frac{11}{6})$

(c) $(\frac{2}{3} \times \frac{3}{8}) + (\frac{2}{3} \times 2\frac{5}{8})$

(d) $(1\frac{3}{5} \times \frac{3}{4}) + (1\frac{3}{5} \times \frac{1}{4})$

(e) $(2\frac{1}{3} \times 1\frac{3}{5}) + (2\frac{1}{3} \times 1\frac{2}{5})$

(f) $(\frac{7}{8} \times \frac{1}{3}) + (\frac{1}{8} \times \frac{1}{3})$

(g) $(\frac{3}{10} \times \frac{1}{2}) + (\frac{17}{10} \times \frac{1}{2})$

(h) $(4\frac{1}{4} \times \frac{3}{8}) + (3\frac{3}{4} \times \frac{3}{8})$

(i) $(1\frac{2}{5} \times 1\frac{1}{3}) + (1\frac{3}{5} \times 1\frac{1}{3})$

2. Use the distributive property to find the products.

(a) $5 \times 6\frac{1}{5}$ (d) $\frac{5}{8} \times 16\frac{2}{5}$ (g) $6\frac{1}{4} \times \frac{1}{2}$

(b) $8 \times 2\frac{3}{4}$ (e) $2\frac{1}{3} \times 6$ (h) $9\frac{3}{8} \times \frac{2}{3}$

(c) $\frac{3}{4} \times 12\frac{1}{3}$ (f) $3\frac{2}{5} \times 10$

3. Find the following products without changing mixed numerals to fractions.

(a) $\begin{array}{r} 4\frac{1}{2} \\ \times\ 1\frac{1}{4} \\ \hline \end{array}$ (b) $\begin{array}{r} 12\frac{1}{3} \\ \times\ 2\frac{3}{4} \\ \hline \end{array}$ (c) $\begin{array}{r} 4\frac{1}{9} \\ \times 6\frac{3}{4} \\ \hline \end{array}$ (d) $\begin{array}{r} 10\frac{1}{4} \\ \times\ 3\frac{4}{5} \\ \hline \end{array}$

(e) $\begin{array}{r} 8\frac{1}{5} \\ \times\ 2\frac{5}{8} \\ \hline \end{array}$ (f) $\begin{array}{r} 15\frac{1}{4} \\ \times\ 4\frac{3}{5} \\ \hline \end{array}$ (g) $\begin{array}{r} 6\frac{3}{5} \\ \times\ 2\frac{2}{3} \\ \hline \end{array}$

SOLUTIONS

SECTION 5 - ACTIVITY

1. Whole numbers are closed for addition (ad and bc are both whole numbers from the previous line.)
2. Whole numbers are closed for multiplication.
3. Definition of multiplication of rational numbers.
4. Whole numbers are closed for multiplication.
5. bd is a non-zero whole number.
6. Definition of a rational number.
7. Multiplication of whole numbers is commutative.
8. $\frac{c}{d} \times \frac{a}{b}$
9. Multiplication of rational numbers is commutative.
10. $\frac{ad + bc}{bd}$
11. Multiplication of whole numbers is commutative.
12. $\frac{cb + da}{db}$
13. Definition of addition of rational numbers.
14. $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$
15. Definition of multiplication of rational numbers.
16. $\frac{a(ce)}{b(df)}$
17. Definition of multiplication of rational numbers
18. $\frac{a}{b} \times (\frac{c}{d} \times \frac{e}{f})$
19. Multiplication of rational numbers is associative.
20. Definition of addition of rational numbers.
21. $\frac{a + 0}{b}$
22. 0 is the identity for addition of whole numbers.
23. $\frac{a}{b} + 0 = \frac{a}{b}$
24. $\frac{cf + de}{df}$
25. Definition of multiplication of rational numbers.
26. Distributive property for whole numbers.
27. Definition of multiplication of rational numbers.
28. Definition of addition of rational numbers.
29. Multiplication of whole numbers is associative.
30. Multiplication of whole numbers is commutative.
31. $(\frac{a}{b} \times \frac{c}{d}) + (\frac{a}{b} \times \frac{e}{f})$
32. $\frac{a \times 0}{b \times 1}$
33. Multiplication property of 0 for whole numbers and 1 is the identity for multiplication of whole numbers.
34. The multiplication property of 0 holds for the rational numbers.

ANSWERS

SECTION 6 - ACTIVITY

1. (a) $\frac{3}{4} \times (\frac{2}{5} + \frac{3}{5}) = \frac{3}{4} \times 1 = \frac{3}{4}$
   (b) $\frac{5}{8} \times (\frac{5}{6} + \frac{11}{6}) = \frac{5}{8} \times 2 = \frac{5}{4}$
   (c) $\frac{2}{3} \times (\frac{3}{8} + 2\frac{5}{8}) = \frac{2}{3} \times 3 = 2$
   (d) $1\frac{3}{5} \times (\frac{3}{4} + \frac{1}{4}) = 1\frac{3}{5} \times 1 = 1\frac{3}{5}$
   (e) $2\frac{1}{3} \times (1\frac{3}{5} + 1\frac{2}{5}) = 2\frac{1}{3} \times 3 = 7$
   (f) $(\frac{7}{8} + \frac{1}{8}) \times \frac{1}{3} = 1 \times \frac{1}{3} = \frac{1}{3}$
   (g) $(\frac{3}{10} + \frac{17}{10}) \times \frac{1}{2} = 2 \times \frac{1}{2} = 1$
   (h) $(4\frac{1}{4} + 3\frac{3}{4}) \times \frac{3}{8} = 8 \times \frac{3}{8} = 3$
   (i) $(1\frac{2}{5} + 1\frac{3}{5}) \times 1\frac{1}{3} = 3 \times 1\frac{1}{3} = 4$

2. (a) $5 \times (6 + \frac{1}{5}) = (5 \times 6) + (5 \times \frac{1}{5}) = 30 + 1 = 31$
   (b) $8 \times (2 + \frac{3}{4}) = (8 \times 2) + (8 \times \frac{3}{4}) = 16 + 6 = 22$
   (c) $\frac{3}{4} \times (12 + \frac{1}{3}) = (\frac{3}{4} \times 12) + (\frac{3}{4} \times \frac{1}{3}) = 9 + \frac{1}{4} = 9\frac{1}{4}$
   (d) $\frac{5}{8} \times (16 + \frac{2}{5}) = (\frac{5}{8} \times 16) + (\frac{5}{8} \times \frac{2}{5}) = 10 + \frac{1}{4} = 10\frac{1}{4}$
   (e) $(2 + \frac{1}{3}) \times 6 = (2 \times 6) + (\frac{1}{3} \times 6) = 12 + 2 = 14$
   (f) $(3 + \frac{2}{5}) \times 10 = (3 \times 10) + (\frac{2}{5} \times 10) = 30 + 4 = 34$
   (g) $(6 + \frac{1}{4}) \times \frac{1}{2} = (6 \times \frac{1}{2}) + (\frac{1}{4} \times \frac{1}{2}) = 3 + \frac{1}{8} = 3\frac{1}{8}$
   (h) $(9 + \frac{3}{8}) \times \frac{2}{3} = (9 \times \frac{2}{3}) + (\frac{3}{8} \times \frac{2}{3}) = 6 + \frac{1}{4} = 6\frac{1}{4}$

3. (a) $\begin{array}{r} 4\frac{1}{2} \\ \times\ 1\frac{1}{4} \\ \hline 1\frac{1}{8} \\ 4\frac{1}{2} \\ \hline 5\frac{5}{8} \end{array}$
   (b) $\begin{array}{r} 12\frac{1}{3} \\ \times\ 2\frac{3}{4} \\ \hline 9\frac{1}{4} \\ 24\frac{2}{3} \\ \hline 33\frac{11}{12} \end{array}$
   (c) $\begin{array}{r} 4\frac{1}{9} \\ \times\ 6\frac{3}{4} \\ \hline 3\frac{1}{12} \\ 24\frac{2}{3} \\ \hline 27\frac{9}{12} \end{array} = 27\frac{3}{4}$
   (d) $\begin{array}{r} 10\frac{1}{4} \\ \times\ 3\frac{4}{5} \\ \hline 8\frac{1}{5} \\ 30\frac{3}{4} \\ \hline 38\frac{19}{20} \end{array}$
   (e) $\begin{array}{r} 8\frac{1}{5} \\ \times\ 2\frac{5}{8} \\ \hline 5\frac{1}{8} \\ 16\frac{2}{5} \\ \hline 21\frac{21}{40} \end{array}$
   (f) $\begin{array}{r} 15\frac{1}{4} \\ \times\ 4\frac{3}{5} \\ \hline 9\frac{3}{20} \\ 61 \\ \hline 70\frac{3}{20} \end{array}$
   (g) $\begin{array}{r} 6\frac{3}{5} \\ \times\ 2\frac{2}{3} \\ \hline 4\frac{2}{5} \\ 13\frac{1}{5} \\ \hline 17\frac{3}{5} \end{array}$

TOPIC II

Post-Test II - Basic (Form A)

Mark from your record page those test-items which you are required to do. Show the work for the required items in the spaces provided. Work carefully and do not spend too much time on any one question.

B 1.1 Find the sums of the following and write each as a basic fraction:

A. $\frac{3}{5} + \frac{5}{6} + \frac{1}{3}$

B. $\begin{array}{r} \frac{3}{4} \\ \frac{1}{12} \\ + \frac{1}{6} \\ \hline \end{array}$

B1.1 - A (______)

B1.1 - B (______)

B 1.2 Find the differences of the following and write each as a basic fraction:

A. $\frac{7}{5} - \frac{4}{3} =$

B. $\begin{array}{r} \frac{16}{7} \\ - \frac{25}{14} \\ \hline \end{array}$

B1.2 - A (______)

B1.2 - B (______)

B 2.1 A. Write the fraction $\frac{39}{9}$ as a mixed numeral.

B. Write $9\frac{3}{4}$ as a fraction.

B2.1 - A (______)

B2.1 - B (______)

B 3.1 Find the sums of the following and write the fraction part in the answer as a basic fraction.

A. $$\begin{array}{r} 11\frac{2}{3} \\ +\ 12\frac{5}{8} \\ \hline \end{array}$$

B. $7\frac{7}{9} + 6\frac{2}{3} + 5\frac{5}{6}$

B3.1 - A (_______)

B3.1 - B (_______)

B3.2 Find the differences of the following and write the fraction part in the answer as a basic fraction.

A. $7\frac{7}{9} - 5\frac{5}{6}$

B. $$\begin{array}{r} 18\frac{3}{7} \\ -\ 14\frac{2}{3} \\ \hline \end{array}$$

B3.2 - A (_______)

B3.2 - B (_______)

B4.1 Solve the condition $a - \frac{13}{14} = \frac{3}{28}$ and write the fraction in the solution as a basic fraction. Show your check.

B4.1 (_______)

TOPIC II

Post-Test II - Intermediate (Form B)

Make sure you check and see if you need a copy of Post-Test II - Basic before you start. Next take your record page and mark all those items which you are required to do. Then do these items and show your work in the spaces provided for this purpose. Work carefully and do not spend too much time on any one question.

I 1.1 Find the sums of the following and write each as a basic fraction.

A. $\frac{7}{5} + \frac{8}{3} + \frac{14}{9}$

B. $\frac{15}{7}$

$\frac{9}{5}$

$+ \frac{3}{35}$

I 1.1 - A (______)

I 1.1 - B (______)

I 1.2 Find the differences of the following and write each as a basic fraction.

A. $\frac{11}{12} - \frac{4}{5}$

B. $\frac{25}{24}$

$- \frac{13}{16}$

I 1.2 - A (______)

I 1.2 - B (______)

I 2.2 A. Justify that the mixed numeral for $\frac{63}{12}$ is $5\frac{1}{4}$ by using fractions.

B. Using fractions justify that the fraction for

$5\frac{1}{3}$ is $\frac{16}{3}$.

I3.2 Find the differences for the following and write the fraction part in each answer as a basic fraction.

A. $45\frac{11}{16} - 15\frac{7}{10}$

B. $17\frac{7}{8}$

$- 3\frac{7}{20}$

I3.2 - A (______)

I3.2 - B (______)

I4.1 Solve the condition $9\frac{9}{16} = m + 3\frac{19}{24}$. Write the fraction in the solution as a basic fraction.
Show check.

I4.1 (______)

TOPIC II

Post-Test II - Advanced (Form A)

Show your work in the spaces provided for this purpose. Work carefully and do not spend too much time on any one question.

A1 Define a non-negative rational number.

A2 Write the definitions for addition and subtraction of rational numbers named by fractions.

A3 Write the definition for multiplication of rational numbers named by fractions.

A4 Write the definition for division of rational numbers named by fractions.

A5 Complete the following proof.

Prove that multiplication is commutative for rational numbers.

Let $\frac{a}{b}$ and $\frac{c}{d}$ be any two rational numbers.

Prove: $\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$

Proof:

Statement	Reason
$\frac{a}{b} \times \frac{c}{d}$	Definition for multiplication of rational numbers
= (1) ____________	
	(2) ____________
$= \frac{ca}{db}$	
$= \frac{c}{d} \times \frac{a}{b}$	(3) ____________
(4) ____________	

i.e. Multiplication of rational numbers is commutative.

A6 A. Show how the distributive property may be used to simplify

$$\frac{7}{15} \times \frac{7}{38} + \frac{4}{5} \times \frac{7}{38}$$

B. Show <u>how the distributive property</u> may be used to find the product

$\frac{3}{8} \times 24\frac{16}{25}$

C. Find the product without changing the mixed numerals to fractions.

$$\begin{array}{r} 8\frac{2}{3} \\ \times\ 2\frac{3}{8} \\ \hline \end{array}$$

CHALLENGERS TOPIC 2

Think back to Problem Solving in Topic 1. It was stated that there are no 'hard-and-fast' rules to follow in the solving of all problems. Each problem will be different and will require its own 'line-of-attack'.

Sometimes a mathematical pattern or relationship can be discovered if we first look at several numerical examples. Here is an example in which looking at examples helps.

The first of two rational numbers is greater than zero, and the product of the two rational numbers is greater than zero.

(I) If the product is less than the first number, what must be true about the second number?

(II) If the product is greater than the first number, what must be true about the second number?

Let's look at some numerical examples to determine what must be true about the second number in each case.

(I)

First Number	Second Number	Product (less than first number)	Finding the Second Number	Second Number
5	n	3	$5 \times n = 3$ $\frac{1}{5} \times 5 \times n = \frac{1}{5} \times 3$ $n = \frac{3}{5}$	$\frac{3}{5}$
7	p	4	$7 \times p = 4$ $\frac{1}{7} \times 7 \times p = \frac{1}{7} \times 4$ $p = \frac{4}{7}$	$\frac{4}{7}$
$\frac{2}{3}$	q	$\frac{1}{4}$	$\frac{2}{3} \times q = \frac{1}{4}$ $\frac{3}{2} \times \frac{2}{3} \times q = \frac{3}{2} \times \frac{1}{4}$ $q = \frac{3}{8}$	$\frac{3}{8}$

What do you notice about all the second numbers?

You can look at as many examples as you require to find a pattern. Here we notice that every time the second number is less than one. Thus, it seems that if the product is less than the first number, then the second number must be less than one.

Now you try some examples for part (II) and see what is true for the second number in each case.

Read the following example very carefully.

'If a given rational number (less than one) is divided by a rational number between 0 and 1, how does the quotient compare in size with the given number?'

Let's try some numerical examples.

Given Number	Number Between 0 and 1	Finding Quotient	Quotient
$\frac{3}{4}$	$\frac{1}{2}$	$\frac{3}{4} \div \frac{1}{2}$ $\frac{3}{4} \times \frac{2}{1}$ $\frac{3}{2}$	$1\frac{1}{2}$
$\frac{3}{5}$	$\frac{4}{15}$	$\frac{3}{5} \div \frac{4}{15}$ $\frac{3}{5} \times \frac{15}{4}$ $\frac{9}{4}$	$2\frac{1}{4}$
$\frac{4}{7}$	$\frac{8}{11}$	$\frac{4}{7} \div \frac{8}{11}$ $\frac{4}{7} \times \frac{11}{8}$ $\frac{11}{14}$	$\frac{11}{14}$

How does the quotient compare in size with the given number? In each case we can see the quotient is greater than the given number, when the given number (less than one) is divided by a rational number between 0 and 1.

Remember that all problems are different. But you may sometimes find it useful to use numerical examples when looking for mathematical relationships or patterns.

The most important thing to do when solving problems is to THINK! THINK! THINK!

Read the following instructions and then do the problems.

1. After trying a problem, check your answer with the one given at the end of the Problem Solving section. If it is incorrect, try the problem again to see if you can arrive at the right answer.
2. Do NOT spend more than 15 to 20 minutes of hard thinking on a single problem. After that time, leave it and try the next problem in the section you are working. You are not expected to do every problem. If you find that the problems in a section are very easy and do not require you to do much thinking, leave this section and start the next section. The problems having been divided into four sections with the easier problems first. For you to be doing real problem solving you should be challenged and be required to think about a problem before being able to arrive at an answer.
3. When you have tried all the problems in a section, return to those you missed. You may be surprised at your second try.
4. Work on some of these problems at home. You may have more time to think about them.
5. Have your teacher look at your "Problem Solving" attempts and achievements.

Challengers Section 1.

1. Suppose you saw the two number sentences below written on the chalkboard by two students. What correct conclusion could you reach about these products <u>without doing any computations</u>? Why?

$$\frac{24}{23} \times \frac{65}{66} = \frac{1560}{1518} \qquad \frac{65}{66} \times \frac{24}{23} = \frac{1580}{1518}$$

2. Joe saw the two sentences below written on the chalkboard. He said "If both sentences are true, then addition of rational numbers is NOT commutative. Was he correct? Explain.

$$\frac{6}{8} + \frac{6}{12} = \frac{30}{24} \qquad \frac{6}{12} + \frac{6}{8} = \frac{5}{4}$$

3. Simplify the following:

a) $\frac{7}{35} + \frac{3}{18} + \frac{8}{105}$

b) $\frac{5}{24} + \frac{7}{30} + \frac{9}{40}$

4. What rational number less than 2 can be added to $\frac{5}{8}$ so that each sum is greater than 1?

5. Show how the distributive property can be used to show that $\frac{1}{2} \times 6\frac{2}{3}$ is $3\frac{1}{3}$.

6. The universe or replacement set for 'n' in the following conditions of equality is the set of whole numbers. Find replacements for 'n' which make each condition true.

a) $\frac{n}{n} = \frac{n}{8}$ b) $\frac{n}{n} = \frac{16}{16}$ c) $\frac{4}{5} \times \frac{3}{n} = \frac{3}{10}$

Challengers Section 2.

7. A certain rational number is greater than zero. Its reciprocal is larger than the number itself. What must be true about the number?

8. What can be said about the value of reciprocals of numbers which are very close to one?

9. If a given rational number (greater than zero) is multiplied by a rational number between 0 and 1, how does the product compare in size with the given number?

10. If a given rational number (greater than zero) is divided by a rational number between 0 and 1, how does the quotient compare in size with the given number?

Challengers Section 3.

11. Express the sum of $\frac{x}{y} + \frac{p}{q}$ as a fraction.

12. If a, b, and c are different whole numbers, find the values of a, b and c if:

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$$

13. Write the solutions for the following sentences. The universe is the set of rational numbers greater than zero.

a) $\frac{3}{4} - n = \frac{1}{8}$

b) $n + \frac{2}{3} > \frac{5}{6}$

c) $n - \frac{2}{5} > \frac{1}{5}$

d) $2n > 3$

e) $\frac{2}{3} n > \frac{1}{6}$

14. Find the following sums. Then find a relationship between each sum and the numbers involved.

a) $\frac{1}{1x2} + \frac{1}{2x3} =$

b) $\frac{1}{1x2} + \frac{1}{2x3} + \frac{1}{3x4} =$

c) $\frac{1}{1x2} + \frac{1}{2x3} + \frac{1}{3x4} + \frac{1}{4x5} =$

d) $\frac{1}{1x2} + \frac{1}{2x3} + \frac{1}{3x4} + \frac{1}{4x5} + \frac{1}{5x6} =$

(cont.)

14. Use this relationship to find the following sums.

e) $\frac{1}{1x2} + \frac{1}{2x3} + \frac{1}{3x4} + \ldots + \frac{1}{49x50} =$

f) $\frac{1}{1x2} + \frac{1}{2x3} + \frac{1}{3x4} + \ldots + \frac{1}{99x100} =$

15. Find the sums of the following fractions and look for a relationship between the sum and the fractions.

eg. $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

a) $\frac{1}{9} + \frac{1}{18} =$ c) $\frac{1}{15} + \frac{1}{30} =$

b) $\frac{1}{12} + \frac{1}{24} =$ d) $\frac{1}{18} + \frac{1}{36} =$

Using the relationship found above, find the following sums. (Do NOT form the L.C.D. and add the fractions, use the relationship.)

e) $\frac{1}{21} + \frac{1}{42} =$ h) $\frac{1}{45} + \frac{1}{90}$

f) $\frac{1}{24} + \frac{1}{48} =$ i) $\frac{1}{48} + \frac{1}{96}$

g) $\frac{1}{30} + \frac{1}{60} =$

Complete the following sentences using the same pattern as above.

j) $\frac{1}{33} + \frac{1}{\square} = \frac{1}{\square}$

k) $\frac{1}{60} + \frac{1}{\square} = \frac{1}{\square}$

l) $\frac{1}{75} + \frac{1}{\square} = \frac{1}{\square}$

m) Write a formula to express the relationship found.

Challengers Section 4.

16. After much experimenting Robert claimed that dividing rational numbers could be done by dividing the numerators and dividing the denominators of the fractions in much the same way as multiplication of rational numbers. Try some numerical examples to determine if Robert was correct. Prove your answer using rational numbers $\frac{a}{b}$ and $\frac{c}{d}$.

17. Demonstrate on a number line the multiplication of the following rational numbers.
a) $\frac{3}{4} \times \frac{8}{9}$ b) $2 \times \frac{4}{3}$ c) $\frac{3}{2} \times \frac{3}{4}$

18. Indicate diagramatically, with regions, how to form the following products.
a) $\frac{2}{3} \times \frac{9}{4}$ b) $\frac{4}{3} \times \frac{3}{4}$ c) $\frac{4}{3} \times \frac{9}{4}$

19. a) Can the distributive property be used to find the product $6\frac{1}{2} \times 2\frac{2}{3}$? If it can, then show how.

b) If you were in the Advanced Group for this topic, prove the distributive property of multiplication over subtraction of rational numbers. (hint: use the definition of subtraction).

20. $$\frac{11}{4} = 2 + \frac{3}{4} = 2 + \frac{1}{\frac{4}{3}} = 2 + \frac{1}{1\frac{1}{3}} = 2 + \frac{1}{1+\frac{1}{3}}$$

$2 + \cfrac{1}{1 + \frac{1}{3}}$ is known as a continued fraction for $\frac{11}{4}$.

It is represented by the symbols (2; 1, 3)

Another example of a continued fraction is:

$$\frac{19}{47} = 0 + \frac{1}{\frac{47}{19}} = 0 + \frac{1}{2\frac{9}{19}} = 0 + \cfrac{1}{2 + \frac{9}{19}} = 0 + \cfrac{1}{2 + \cfrac{1}{\frac{19}{9}}} = 0 + \cfrac{1}{2 + \cfrac{1}{2\frac{1}{9}}} = 0 + \cfrac{1}{2 + \cfrac{1}{2+\frac{1}{9}}}$$

The continued fraction is represented by (0; 2, 2, 9).

* Each numerator in a continued fraction is one.

a) Compute the continued fraction for $\frac{64}{15}$ and write the symbol for it.

b) Which fraction is represented by the continued fraction (0; 1, 2, 3)?

CHALLENGERS ANSWERS Topic 2

Discussion:

If the product is greater than the first rational number, the second rational number must be greater than one.

Section 1

1. One of the products must be incorrect. According to the commutative property of multiplication for rational numbers, they should have equal products.
2. No, he was not correct.

$$\frac{30}{24} = \frac{5}{4} \quad \text{ie. } \frac{6}{8} + \frac{6}{12} = \frac{6}{12} + \frac{6}{8}$$

Thus the commutative property of addition for rational numbers is supported.

3. a) $\frac{31}{70}$ b) $\frac{2}{3}$
4. $\frac{3}{8} < n < 2$ ie. the set of rational numbers from $\frac{3}{8}$ to 2.
5. $\frac{1}{2} \times 6\frac{2}{3} = \frac{1}{2} \times (6 + \frac{2}{3}) = (\frac{1}{2} \times 6) + (\frac{1}{2} \times \frac{2}{3}) = 3 + \frac{1}{3} = 3\frac{1}{3}$.
6. a) 8 b) any whole number greater than zero c) 8

Section 2

7. The number is less than one.
8. The value of the reciprocals is also very close to one.
9. The product is smaller than the given number.
10. The quotient is greater than the given number.

Section 3

11. $\frac{xq + py}{yq}$ 12. $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$
13. a) sol: $\frac{5}{8}$ b) sol: all rational numbers greater than $\frac{1}{6}$.

    c) sol: all rational numbers greater than $\frac{3}{5}$.

    d) sol: all rational numbers greater than $\frac{3}{2}$.

    e) sol: all rational numbers greater than $\frac{1}{4}$.

14. a) $\frac{2}{3}$ b) $\frac{3}{4}$ c) $\frac{4}{5}$ d) $\frac{5}{6}$ e) $\frac{49}{50}$ f) $\frac{99}{100}$

15. a) $\frac{1}{6}$ b) $\frac{1}{8}$ c) $\frac{1}{10}$ d) $\frac{1}{12}$ e) $\frac{1}{14}$ f) $\frac{1}{16}$ g) $\frac{1}{20}$

h) $\frac{1}{30}$ i) $\frac{1}{32}$ j) $\frac{1}{33} + \frac{1}{66} = \frac{1}{22}$ k) $\frac{1}{60} + \frac{1}{120} = \frac{1}{40}$

l) $\frac{1}{75} + \frac{1}{150} = \frac{1}{50}$ m) $\frac{1}{3n} + \frac{1}{6n} = \frac{1}{2n}$

Section 4

16. Robert was correct.

Usual method $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

$= \frac{a \times d}{b \times c}$

Robert's method $\frac{a}{b} \div \frac{c}{d} = \frac{a \div c}{b \div d}$

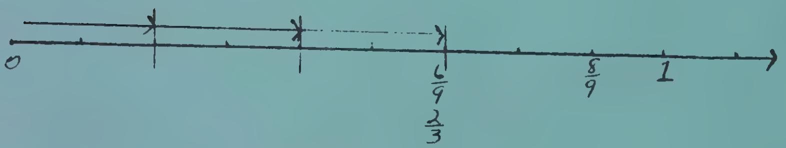
$= \dfrac{\frac{a}{c}}{\frac{b}{d}}$

Robert's method gives the same answer as the usual method.

$= \frac{a}{c} \times \frac{d}{b}$

$= \frac{a \times d}{c \times b}$

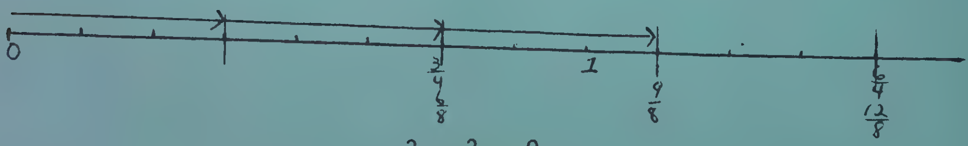
$= \frac{a \times d}{b \times c}$

17. a) 

$\frac{3}{4} \times \frac{8}{9} = \frac{2}{3}$

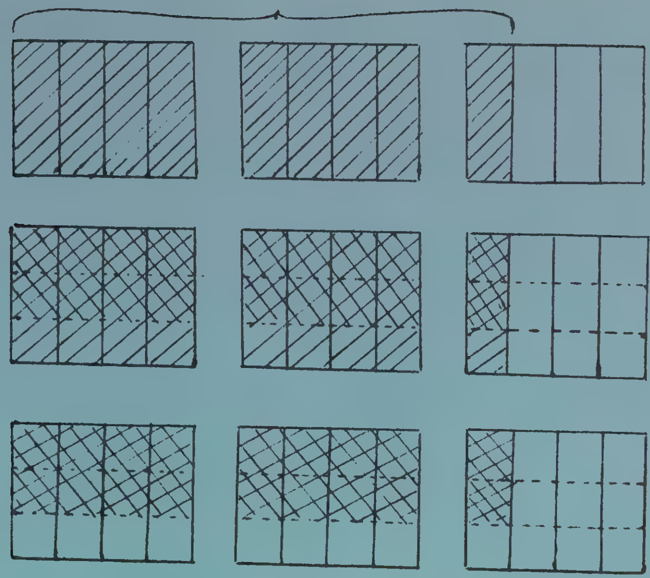
b) 

$2 \times \frac{4}{3} = \frac{8}{3}$

c) 

$\frac{3}{2} \times \frac{3}{4} = \frac{9}{8}$

18. a)

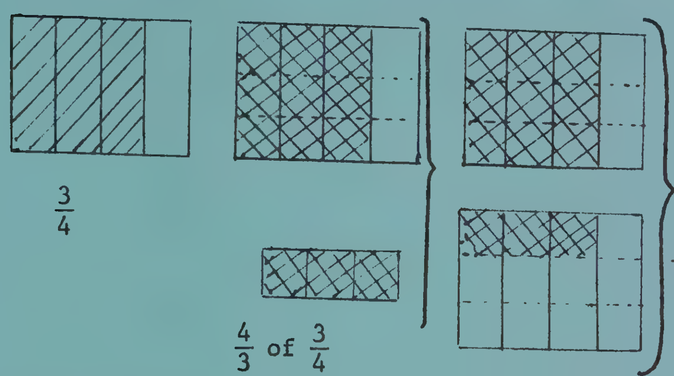


$\frac{9}{4}$

$\frac{2}{3}$ of $\frac{9}{4}$

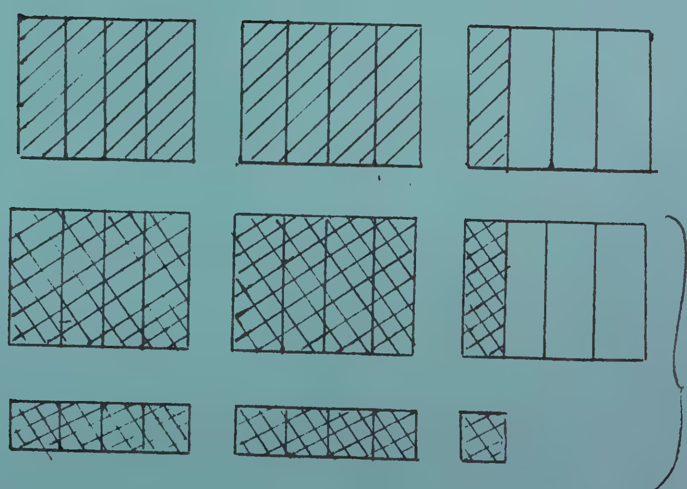
$\frac{2}{3}$ of $\frac{9}{4} = \frac{18}{12}$

b)



$\frac{4}{3}$ of $\frac{3}{4} = \frac{12}{12}$

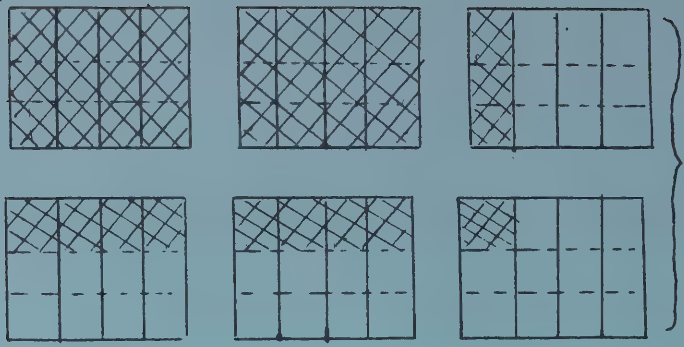
c)



$\frac{9}{4}$

$\frac{4}{3}$ of $\frac{9}{4}$

cont. c)



$\frac{4}{3}$ of $\frac{9}{4} = \frac{36}{12}$

19. a) $6\frac{1}{2} \times 2\frac{2}{3} = 6\frac{1}{2} \times (2 + \frac{2}{3})$

$$= (6\frac{1}{2} \times 2) + (6\frac{1}{2} \times \frac{2}{3})$$

$$= (6 + \frac{1}{2}) \quad \times \quad 2 \quad + (6 \times \frac{1}{2}) \times \frac{2}{3}$$

$$= (6 \times 2) + (\frac{1}{2} \times 2) + (6 \times \frac{2}{3}) + (\frac{1}{2} \times \frac{2}{3})$$

$$= \quad 12 \quad + \quad 1 \quad + \quad 4 \quad + \quad \frac{1}{3}$$

$$= 17\frac{1}{3}$$

b) Let $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ be any rational numbers.

Prove: $\frac{a}{b} \times (\frac{c}{d} - \frac{e}{f}) = (\frac{a}{b} \times \frac{c}{d}) - (\frac{a}{b} \times \frac{e}{f})$

Proof:

Statement	Reason	Comments
$\frac{a}{b} \times (\frac{c}{d} - \frac{e}{f})$		Start with left side.
$= \frac{a}{b} \times (\frac{cf - de}{df})$	Definition of subtraction of rational numbers.	
$= \frac{a \times (cf - de)}{b \times (df)}$	Definition of multiplication of rational numbers.	
$= \frac{acf - ade}{bdf}$	Distributive property of whole numbers.	cf and de are whole numbers.
$(\frac{a}{b} \times \frac{c}{d}) - (\frac{a}{b} \times \frac{e}{f})$		Now start with right side and get it to be the same as the fourth line.
$= \frac{ac}{bd} - \frac{ae}{bf}$	Definition of multiplication of rational numbers.	
$= \frac{acbf - aebd}{bdbf}$	Definition of subtraction of rational numbers.	

Statement	Reason	Comments
$= \dfrac{b(acf - dae)}{bdbf}$	Distributive property of whole numbers.	
$= \dfrac{\cancel{b}(acf - dae)}{\cancel{b}dbf}$	Reducing the fraction.	
$= \dfrac{acf - ade}{bdf}$	Commutative property of whole numbers	Order is changed in some terms.
$\therefore \frac{a}{b} \times (\frac{c}{d} - \frac{e}{f}) = (\frac{a}{b} \times \frac{c}{d}) - (\frac{a}{b} \times \frac{e}{f})$		Result is same as left side.

ie. Multiplication is distributive over subtraction of rational numbers.

20. a) $\frac{64}{15} = 4 + \frac{4}{15}$

$$= 4 + \cfrac{1}{\frac{15}{4}}$$

$$= 4 + \cfrac{1}{3\frac{3}{4}}$$

$$= 4 + \cfrac{1}{3 + \frac{3}{4}}$$

$$= 4 + \cfrac{1}{3 + \cfrac{1}{\frac{4}{3}}}$$

$$= 4 + \cfrac{1}{3 + \cfrac{1}{1\frac{1}{3}}}$$

$$= 4 + \cfrac{1}{3 + \cfrac{1}{1 + \frac{1}{3}}}$$

$\therefore$ (4; 3, 1, 3)

b) $(0; 1, 2, 3) = 0 + \cfrac{1}{1 + \cfrac{1}{2 + \frac{1}{3}}}$

$$= 0 + \cfrac{1}{1 + \cfrac{1}{2\frac{1}{3}}}$$

$$= 0 + \cfrac{1}{1 + \cfrac{1}{\frac{7}{3}}}$$

$$= 0 + \cfrac{1}{1 + \frac{3}{7}}$$

$$= 0 + \cfrac{1}{1\frac{3}{7}}$$

$$= 0 + \cfrac{1}{\frac{10}{7}}$$

$$= 0 + \frac{7}{10}$$

$\therefore$ the fraction is $\frac{7}{10}$.

APPENDIX B

THE OBSERVATION SCHEDULE

OBSERVATION SCHEDULE

Tot	1.0 CHK ASSM	I	III	V
	1.1 t wks ex at bd			
	1.2 t rds hw answ			
	1.3 t asks for answ			
	1.4 t chks ind hw			
	1.5 t cmmnts on hw			
	1.6 p wrks ex at bd			
	1.7 p hnds in hw			
	2.0 FORM TCH			
	2.1 t lects			
	2.2 t uses bd			
	2.3 t uses mp, cht, etc.			
	2.4 t uses sld,flm,etc.			
	2.5 t uses aud aid			
	2.6 t uses obj			
	2.7 t uses sp t aid			
	2.8 t uses no mat			
	2.9 t qu, p ans			
	2.10 t ans p qu			
	2.11 t ign p qu			
	2.12 t ref p qu			
	2.13 t lds disc			
	2.14 t rvws prev wrk			
	2.15 t uses hnd out			
	2.16 t retchs			
	2.17 p dem at bd			
	2.18 p lds clss disc			
	2.19 p wrks ex at st			
	2.20 p wrks ex at bd			
	3.0 DIR PRACT			
	3.1 t expl			
	3.2 t chks ex wrkd			
	3.3 t retchs idea			
	3.4 t hlps ind			
	3.5 t wrks prbl			
	3.6 p wrks ex at st			
	3.7 p dem at bd			
	3.8 p hlps p			
	3.9 p tlks to sm grp			
	3.10 t qu, p ans			

SUBJ. OF LSSN TGHT:

NO. PUPILS IN CLASS:

TCHR. OBS.:

DATE:

4.0 MANGRL TSKS	I	III	V	Tot
4.1 t tks attnd				
4.2 t clns bd				
4.3 t discpl				
4.4 t mrks assm				
4.5 t pss out mat				
4.6 t wrks at dsk				
4.7 t ans dr/phn				
4.8 t lvs/ents rm				
4.9 t wrts on bd				
4.10 t mks assm				
4.11 t mks annomnt				
4.12 p clns bd				
4.13 p pss out mat				
4.14 p clns rm				
4.15 p ans dr/phn				
4.16 t tlls p to hlp p				
5.0 TESTING				
5.1 t supv tst				
5.2 t supv quz				
5.3 p wrts tst/quz				
5.4 cls maks/tks up tst				
5.5 t ign p qu				
5.6 t ans p qu				
6.0 ST WRK				
6.1 t hlps ind				
6.2 t wks w sm gp				
6.3 t urg p to wrk				
6.4 p st, rds at st				
6.5 p wrts, mnps at st				
6.6 p clrs,drws,cts,etc.				
6.7 p asks qu, t hlps				
6.8 p uses hnd out				
6.9 p uses txt				
6.10 p uses wrkbk				
6.11 p uses auth rdg				
6.12 p hlps p				
6.13 t expls to cls				
6.14 t chks prbl wrkd				
7.0 LAB WORK				
7.1 t drcts p to lab				
7.2 p uses pzz, gms				
7.3 p uses suppl rd				
7.4 p uses calc, etc.				

SECOND PAGE OF OBSERVATION SCHEDULE

				Soc Adm				
Tot	I	III	V	8.0 GROUPG	I	III	V	Tot
				8.1 at 1st ½ cl in gp w t				
				8.2 at 1st ½ cl in gp w/o t				
				8.3 4 p to ½ cl in gp w t				
				8.4 4 p to ½ cl in gp w/o t				
				8.5 2-3 p in gp w t				
				8.6 2-3 p in gp w/o t				
				8.7 p as ind				

9.0 CLSRM CLIM	I	III	V	Tot
9.1 t mvs frly				
9.2 t discpl p dir				
9.3 t shws afct f p				
9.4 t shws host t p				
9.5 t uses sarcsm				
9.6 t yls				
9.7 p mvs frly				
9.8 p shws afct f t				
9.9 p shws host t t				
9.10 p shws host t p				
9.11 p ign t qu				
9.12 p scfls, frts				
9.13 p wsps				
9.14 p lghs				
9.15 p lvs/entrs rm				
9.16 p l st				
9.17 nse and tme wstng				

REMARKS:

9.18 p works at bd

OBSERVATION OF AFFECTIVE TEACHER	II		IV		VI			Tot
TALK AND ACTIONS 10.0	IND	GP	IND	GP	IND	GP	IND	GP
10.1 t apprvs, aft gest, lk								
10.2 mks p supportv stmnt								
10.3 mks prblm strctg stmnt								
10.4 mks misc stmnt								
10.5 mks drtve stmnt								
10.6 mks reprvng stmnt								
10.7 reprvng host gest, lk								
10.8 t assrts authrty								

APPENDIX C

THE LOG SHEET

APPENDIX C

TEACHER:

DALY LOG FOR WEEK FROM TO FOR GRADE SEVEN MATHEMATICS

INDICATE IN THIS SECTION TOTAL TIME DEVOTED TO GRADE SEVEN MATHEMATICS AS INDICATED	TIME SPENTON PLANNING LESSON FOR					TIME SPENTON PREPARING AIDS FOR					TIMESPENTON MARKING ASSIGNMENTS FOR					TIMESPENTON PREPARING TESTS FOR					TIMESPENTON MARKING TESTS FOR					TIME SPENT ON RECORD-KEEPING FOR					TIME SPENT ON DEPARTMENTAL MEETING FOR					TIME SPENT ON GROUP PLANNING FOR				
	1-15	15-30	30-45	45-60	OVER 60	1-15	15-30	30-45	45-60	OVER 60	1-15	15-30	30-45	45-60	OVER 60	1-15	15-30	30-45	45-60	OVER 60	1-15	15-30	30-45	45-60	OVER 60	1-15	15-30	30-45	45-60	OVER 60	1-15	15-30	30-45	45-60	OVER 60	1-15	15-30	30-45	45-60	OVER 60
WEEKEND																																								
MONDAY																																								
TUESDAY																																								
WEDNESDAY																																								
THURSDAY																																								
FRIDAY																																								

CLASS#1 – IF YOU TEACH MORE THAN ONE CLASS REPORT FOR EACH THE FOLLOWING–	TIME SPENT ON TEACHING ENTIRE CLASS					TIME SPENT ON TEACHING DIFF. ABILITY GROUPS					TIME SPENT ON TEACHING SMALL GROUPS (3-6)					TIME SPENT ON HELPING INDIV. PUPILS IN CLASS					TIME SPENT ON SUPERVISING TEST/QUIZZ					TIME SPENT ON TAKING UP HOME WORK					CLASS SPENT THIS TIME AT SEAT WORK					CLASS SPENT THIS TIME AT WORKING SAMPLE AT SEAT/BBOARD				
	1-10	10-20	20-30	30-40	40-50	1-10	10-20	20-30	30-40	40-50	1-10	10-20	20-30	30-40	40-50	1-10	10-20	20-30	30-40	40-50	1-10	10-20	20-30	30-40	40-50	1-10	10-20	20-30	30-40	40-50	1-10	10-20	20-30	30-40	40-50	1-10	10-20	20-30	30-40	40-50
MONDAY																																								
TUESDAY																																								
WEDNESDAY																																								
THURSDAY																																								
FRIDAY																																								

CLASS #2	TIME SPENT ON TEACHING ENTIRE CLASS					TIME SPENT ON TEACHING DIFF. ABILITY GROUPS					TIME SPENT ON TEACHING SMALL GROUPS (3-6)					TIME SPENTON HELPING INDIV. PUPILS IN CLASS					TIME SPENT ON SUPERVISING TEST/QUIZZ					TIME SPENTON TAKING UP HOME WORK					CLASS SPENT THIS TIME AT SEAT WORK					CLASS SPENT THIS TIME AT WORKING SAMPLE AT SEAT/BBOARD				
	1-10	10-20	20-30	30-40	40-50	1-10	10-20	20-30	30-40	40-50	1-10	10-20	20-30	30-40	40-50	1-10	10-20	20-30	30-40	40-50	1-10	10-20	20-30	30-40	40-50	1-10	10-20	20-30	30-40	40-50	1-10	10-20	20-30	30-40	40-50	1-10	10-20	20-30	30-40	40-50
MONDAY																																								
TUESDAY																																								
WEDNESDAY																																								
THURSDAY																																								
FRIDAY																																								

APPENDIX D

THE QUESTIONNAIRE

APPENDIX D

Questionnaire

The purpose of this questionnaire is to find out the difference, if any in the nature of the activities teachers have to perform in the regular classroom setting and those activities to be performed in the experimental classroom setting. May I ask a few minutes of your time to assist me in obtaining part of this information? In each question circle the answer which best describes your own situation. Answer all questions in relation to the teaching of grade seven mathematics. Answer as accurately as possible. Thank you.

Please supply the following information for the purpose of group comparisons.

Degree(s) held: ______________________________

Latest teaching certificate held: ______________________________

Years of teaching experience: ______________________________

Major field of study: ______________________________

Number of years taught at the grade 7 level: ______________________________

1. Do you plan the units of grade 7 mathematics mostly alone?	1.	yes	no
2. Do you always plan the units of grade seven mathematics mostly with other members of the department?	2.	yes	no
3. Is a unit of grade 7 mathematics usually planned with the entire department?	3.	yes	no

4. Are the unit plans usually prepared

by the Department Head for all the teachers? 4. yes no

5. Are the unit plans usually prepared by a single teacher other than the Department Head for the entire department? 5. yes no

6. Are the unit plans usually prepared by a committee of teachers for the entire department? 6. yes no

7. Do you usually plan the lessons for a grade 7 class alone? 7. yes no

8. Do you usually plan the lessons for a grade 7 math. class completely in cooperation with another member of the department? 8. yes no

9. Do you usually plan the lessons for a grade 7 math. class in cooperation with the entire department? 9. yes no

10. Are the lessons usually planned together with the pupils? 10. yes no

11. Do you usually teach the class assigned to you for the entire period? 11. yes no

12. Do you usually teach the assigned class for most of the period? 12. yes no

13. Do you usually teach the assigned class in cooperation with another teacher? 13. yes no

14. Do you usually teach the assigned class in cooperation with all the other grade 7 mathematics teachers?	14.	yes	no
15. Do you teach most lessons usually to the entire class at once?	15.	yes	no
16. Do you divide your class into 2 or 3 groups (according to ability) and teach each group separately?	16.	yes	no
17. Do you teach most lessons by having pupils working in small groups on prepared materials?	17.	yes	no
18. Do you teach most lessons by having pupils work independently on prepared materials?	18.	yes	no
19. Do you teach most lessons by having the pupils work in small groups or independently using prescribed sections in a textbook(s)?	19.	yes	no
20. Do you expect all pupils to progress through the program at the same rate?	20.	yes	no
21. Do you expect all pupils of the same ability to progress through the program at the same rate?	21.	yes	no
22. Do you expect all pupils to progress at their own rate?	22.	yes	no
23. In your department do you teach as individual teachers (independently)?	23.	yes	no

24. In your department do you teach as a team? 24. yes no

25. Do you have the assistance of a teacher aide? 25. yes no

If the answer to question 25 is yes, then please answer questions 26 - 32.

26. Does the teacher aide assist the teachers of grade 7 mathematics:

(a) ½ day or less per week?

(b) about 1 day per week?

(c) about 3 half days per week?

(d) about 2 days per week?

(e) more than 2 days per week? 26. a b c d e

27. Does the teacher aide supervise test periods? 27. yes no

28. Does the teacher aide mark tests from prepared keys? 28. yes no

29. Does the teacher aide enter marks and keep records? 29. yes no

30. Does the teacher aide perform clerical duties? 30. yes no

31. Does the teacher aide assist in the handling of aids and equipment? 31. yes no

32. Does the teacher aide assist in supervision of classes? 32. yes no

33. Do you get to know individual pupils better or equally well in the individualized study setting as compared to the regular teacher taught class? 33. better equally well

34. Do you find that you discover weaknesses faster or as quickly in the individualized study class as compared to the regular teacher taught class? 34. faster as quickly

35. Has the individualized teaching method influenced your regular classroom teaching? 35. yes no

36. Do you find that in your regular classroom teaching you use:
   (a) some ideas from the experimental method of teaching?
   (b) a fair number of ideas from the experimental method?
   (c) a large number of ideas from the experimental method? 36. a b c

37. Have you become more aware of individual differences since you became involved in individualized instruction? 37. yes no

38. Are you no more aware of individual differences now than you were before you became involved in individualized instruction? 38. yes no

39. Do you try to do more for individual students in the regular teacher taught class since you have become involved in individualized instruction? 39. yes no

40. For individual students in the regular teacher taught class do you do no more now than you did before you became involved in individualized instruction? 40. yes no

41. Do you prefer regular teacher taught classes over individualized instruction? 41. yes no

42. Do you prefer individualized instruction over regular teacher taught classes? 42. yes no

43. Do you have no preference with respect to regular teacher taught classes or individualized instruction? 43. yes no

44. Do you find that individualized instruction helps you to do more for individual pupils in a class than you can do in the regular classroom instruction? 44. yes no

45. Do you find that individualized instruction helps you to do more for small groups of pupils with common problems than you can do in regular classroom instruction? 45. yes no

46. Do you find that individualized instruction helps you to do more for the class as a whole than you can do in regular classroom instruction? 46. yes no

47. Do you find that individualized instruction does more for the above average students than regular instruction does? 47. yes no

48. Do you find that individualized instruction does more for the average student than regular instruction does? 48. yes no

49. Do you find that individualized instruction does more for the poorer student than regular instruction does? 49. yes no

50. Do you find that individualized instruction slows the progress of the above average student? 50. yes no

51. Do you find that individualized instruction slows the progress of the average student? 51. yes no

52. Do you find that individualized instruction slows the progress of the below average student? 52. yes no

53. Do you find that discipline is more
difficult to maintain in

a. the individualized instruction class than in the regular taught class

b. the regular taught class than in the individualized instruction class

c. no difference 53. a b c

54. List aspects about individualized instruction which you do not like:

55. List aspects about individualized instruction which you do like:

APPENDIX E

TABULATION TABLES FOR DATA OBTAINED FROM THE OBSERVATION SCHEDULE

CHECKING ASSIGNMENTS

Teacher Behavior	CONTROL 1	2	3	Tot.	%	INDEPENDENT-STUDY 4	5	6	7	8	Tot.	%	TEACHER-TAUGHT 4	5	6	7	8	Tot.	%
1.1			1	1	3						0	0						0	0
1.2	1			1	3						0	0						0	0
1.3	1	1	2	4	12						0	0						0	0
1.4	1		2	3	9						0	0						0	0
1.5	3		2	5	15	1			1		2	4						0	0
1.6	2			2	6						0	0						0	0
1.7				0	0						0	0						0	0
					33							45							27

FORMAL TEACHING

Teacher Behavior	CONTROL 1	2	3	Tot.	%	INDEPENDENT-STUDY 4	5	6	7	8	Tot	%	TEACHER-TAUGHT 4	5	6	7	8	Tot.	%
2.1	2	2	7	11	33		1		1	1	3	7	6	6	4	6	3	25	93
2.2	2	4	6	12	36						0	0	5	6	6	6	3	26	96
2.3				0	0		1				1	2						0	0
2.4				0	0						0	0						0	0
2.5				0	0						0	0						0	0
2.6				0	0						0	0	1					1	4
2.7				0	0						0	0						0	0
2.8		4	2	6	18						0	0						0	0
2.9	2	5	7	14	42				1	2	3	7	5	6	6	5	3	25	93
2.10	1	6		7	21		1				1	2	1	2	1	2		6	22
2.11	1	2		3	9		1				1	2				1		1	4
2.12			1	1	3						0	0						0	0
2.13	2	1	4	7	21	3					3	7	2		5		1	8	30
2.14		3	2	5	15						0	0	2	2	2	2	1	9	33
2.15				0	0						0	0						0	0
2.16		2	1	3	9						0	0			1			1	4
2.17		2		2	6						0	0						0	0
2.18				0	0						0	0						0	0
2.19		1		1	3						0	0					1	1	4
2.20		2		2	6						0	0					1	1	4
					33							45							27

DIRECTED PRACTISE

Teacher Behavior	CONTROL					INDEPENDENT-STUDY							TEACHER-TAUGHT						
	1	2	3	Tot.	%	4	5	6	7	8	Tot.	%	1	2	3	4	5	Tot	%
3.1		1	2	3	9						0	0						0	0
3.2		1	3	4	12						0	0						0	0
3.3		1	1	2	6						0	0						0	0
3.4			1	1	3						0	0						0	0
3.5		1	3	4	12						0	0						0	0
3.6		1	4	5	15						0	0						0	0
3.7			1	1	3						0	0						0	0
3.8			1	1	3						0	0						0	0
3.9				0	0						0	0						0	0
3.10			1	1	3						0	0						0	0
					/33							/45							/27

MANAGERIAL TASKS

Teacher Behavior	CONTROL					INDEPENDENT-STUDY							TEACHER-TAUGHT						
	1	2	3	Tot.	%	4	5	6	7	8	Tot.	%	4	5	6	7	8	Tot.	%
4.1				0	0						0	0						0	0
4.2	4		1	5	15						0	0	4	3	2	2	2	13	48
4.3	8	2	6	16	49	1	1	5	3	1	11	24	2		2			4	15
4.4			1	1	3				1		1	2						0	0
4.5	2		1	3	9		4	4	1	1	10	22						0	0
4.6				0	0	2	3	4		6	15	33						0	0
4.7	1			1	3	1		3	3	2	9	20						0	0
4.8				0	0	4	4	2		1	11	24						0	0
4.9			2	2	6					1	1	2						0	0
4.10		1	2	3	9				1		1	2					1	1	4
4.11		2	3	5	15	3	5	3	2	4	17	38	4		1	2	1	8	30
4.12				0	0						0	0			1			1	4
4.13				0	0	1			1		2	4						0	0
4.14				0	0						0	0						0	0
4.15	1			1	3				2		2	4						0	0
4.16				0	0					1	1	2						0	0
					/33							/45							/27

TESTING

	CONTROL					INDEPENDENT-STUDY							TEACHER-TAUGHT						
Teacher Behavior	1	2	3	Tot.	%	4	5	6	7	8	Tot.	%	4	5	6	7	8	Tot.	%
5.1				0	0			5			5	11						0	0
5.2				0	0						0	0						0	0
5.3				0	0		6	8	3	9	26	58						0	0
5.4	6			6	18						0	0						0	0
5.5	6			6	18						0	0						0	0
5.6	6			6	18						0	0						0	0
				/33							/45							/27	

SEAT WORK

	CONTROL					INDEPENDENT-STUDY							TEACHER-TAUGHT						
Teacher Behavior	1	2	3	Tot.	%	4	5	6	7	8	Tot.	%	4	5	6	7	8	Tot.	%
6.1	1	3	2	6	18	3	2	6	8	8	27	60						0	0
6.2				0	0	1		1		1	3	7						0	0
6.3	1		2	3	9			4	1	4	9	20						0	0
6.4			1	1	3	6	3	7	8	8	32	71	1					1	4
6.5	1	2	2	5	15	6	4	8	8	8	34	76	1					1	4
6.6				0	0		1				1	2						0	0
6.7	1		1	2	6	4	1	2	2	4	13	29						0	0
6.8				0	0		3		3		6	13						0	0
6.9	1	2	3	6	18						0	0						0	0
6.10				0	0	6	1	7	8	8	30	67	1					1	4
6.11				0	0						0	0						0	0
6.12			2	2	6	5	1	2	2	9	19	42						0	0
6.13		1		1	3						0	0						0	0
6.14		2		2	6						0	0						0	0
				/33							/45							/27	

LABORATORY WORK

	CONTROL					INDEPENDENT-STUDY							TEACHER-TAUGHT						
Teacher Behavior	1	2	3	Tot.	%	4	5	6	7	8	Tot.	%	4	5	6	7	8	Tot.	%
7.1				0	0		1			1	2	4						0	0
7.2	1			1	3		2				2	4						0	0
7.3				0	0						0	0						0	0
7.4				0	0						0	0						0	0

33 45 27

GROUPING (ADMINISTRATIVE)

	CONTROL					INDEPENDENT-STUDY							TEACHER-TAUGHT						
Teacher Behavior	1	2	3	Tot.	%	4	5	6	7	8	Tot.	%	4	5	6	7	8	Tot.	%
8.1	12	8	10	30	91	5	7	3	1	3	19	42	6	6	6	6	3	27	100
8.2				0	0						0	0						0	0
8.3				0	0	1			2	1	4	9						0	0
8.4				0	0						0	0						0	0
8.5				0	0						0	0						0	0
8.6			1	1	3	1					1	2						0	0
8.7		2	2	4	12	3	1		1		5	11						0	0

33 45 27

GROUPING (SOCIAL)

	CONTROL					INDEPENDENT-STUDY							TEACHER-TAUGHT						
Teacher Behavior	1	2	3	Tot.	%	4	5	6	7	8	Tot.	%	4	5	6	7	8	Tot.	%
8.1				0	0						0	0						0	0
8.2				0	0						0	0						0	0
8.3				0	0			2			2	4						0	0
8.4				0	0						0	0						0	0
8.5				0	0						0	0						0	0
8.6				0	0	3		6	2	7	18	40						0	0
8.7				0	0	3	1	7	6	8	25	56						0	0

33 45 27

CLASSROOM CLIMATE

	CONTROL					INDEPENDENT-STUDY							TEACHER-TAUGHT						
Teacher Behavior	1	2	3	Tot.	%	4	5	6	7	8	Tot.	%	4	5	6	7	8	Tot.	%
9.1	11	5	12	28	85	2	5	1	7	4	19	42	4	6	4	4	3	21	78
9.2	5	4	7	16	49	1		2	1	3	7	16	3			2	2	7	26
9.3	2	3	5	10	30	5	3	6	6	7	27	60	4	4	5	3	1	17	63
9.4	1		1	2	6						0	0						0	0
9.5				0	0						0	0						0	0
9.6	2			2	6						0	0						0	0
9.7	1	2		3	9	5	3	6	8	7	29	64						0	0
9.8	2		1	3	9	2	1	3	4	4	14	31				2		2	7
9.9			1	1	3						0	0						0	0
9.10				0	0				1		1	2						0	0
9.11			1	1	3						0	0						0	0
9.12	2	3	1	6	18	1			3		4	9			1			1	4
9.13	12	7	11	30	91	7	4	9	8	9	37	82	1		2	1	2	6	22
9.14	1			1	3	2	1	2	2	3	10	22	3			1		4	15
9.15	2	2	2	6	18	2	4	5	3	3	17	38						0	0
9.16		2	1	3	9	1				1	2	4						0	0
9.17	11	1		12	36	4	3	9	8	9	33	73						0	0
9.18				0	0			3	1		4	9						0	0
					33							45							27

AFFECTIVE TEACHER TALK AND ACTIONS (GROUP)

	CONTROL					INDEPENDENT-STUDY							TEACHER-TAUGHT						
Teacher Behavior	1	2	3	Tot.		4	5	6	7	8	Tot.		4	5	6	7	8	Tot.	
10.1	2	1	8	11	67		3		1		4	2	10	5	15	8	3	41	30
10.2	30	19	37	86	52	2	3		2		7	3	18	9	14	19	4	64	47
10.3	29	26	65	120	73		1				1	0	58	34	40	38	34	204	151
10.4	101	45	70	216	131	52	48	2	4		106	47	24	50	38	61	7	180	133
10.5	67	33	51	151	92	3	19	3	4		29	13	26	32	20	42	10	130	96
10.6	12	7	12	31	19	4	5		2		11	5	1	2	3	1	5	12	9
10.7	14	2	1	17	10	3	2	2	3		10	4	1		2	1		4	3
10.8	22	10	8	40	24	5	2	3	1		11	5	3		4	3	2	12	9
					165							225							135

AFFECTIVE TEACHER TALK AND ACTIONS (INDIVIDUAL)

	CONTROL					INDEPENDENT-STUDY							TEACHER-TAUGHT						
Teacher Behavior	1	2	3	Tot.	Per 100 min.	4	5	6	7	8	Tot.	Per 100 min	4	5	6	7	8	Tot	Per 100 min.
10.1		3	7	10	6	2	13	31	24	29	99	44	2					2	2
10.2		4	11	15	9	9	2	1		7	19	8	5					5	4
10.3		8	13	21	13	12	1			3	16	7	3					3	2
10.4		6	16	22	13	14	2	9	4	14	43	19	3					3	2
10.5		8	10	18	11	14	1	4		15	34	15	2					2	2
10.6		1	5	6	4	2		6		1	9	4						0	0
10.7			1	1	1		1	4	2		7	3	1					1	1
10.8		1	2	3	2	1	3	7	6	5	22	10	2					2	2

/165 /225 /135

APPENDIX F

TABULATION TABLES FOR THE DATA OBTAINED FROM THE LOG SHEET

APPENDIX F

TIME SPENT ON PREPARING TESTS													
CONTROL						EXPERIMENTAL							
Time Interval \ Teacher	1	2	Total	Av/W T	Time W/T	4	5	6	7	8	Tot.	Av/ W/T	Time/ W/T
1 - 15	2	-	2	0.5	3.8	-	-	-	-	-	-	-	-
15 - 30	2	-	2	0.5	11.3	-	-	-	-	-	-	-	-
30 - 45	-	-	-	-	-	-	-	-	-	-	-	-	-
45 - 60	-	1	1	0.25	13.1	-	-	-	-	-	-	-	-
over 60	-	-	-	-	-	-	-	-	-	-	-	-	-
	Divisor		4	Tot. in Min.	28					Divisor	15	Tot. in Min.	0

TIME SPENT ON HELPING INDIVIDUAL PUPILS IN CLASS													
CONTROL						EXPERIMENTAL							
Time Interval \ Teacher	1	2	Tot.	Av/ Class	Time/ Cl.	4	5	6	7	8	Tot.	Av/ Class	Time/ Cl.
1 - 10	8	3	11	0.41	2.1	1	24	6	-	7	38	0.35	1.8
10 - 20	-	3	3	0.11	1.7	13	-	4	-	1	18	0.17	2.6
20 - 30	-	-	-	-	-	8	-	4	6	2	20	0.19	4.8
30 - 40	-	-	-	-	-	-	-	8	4	-	12	0.11	3.9
40 - 50	-	2	2	0.07	3.2	-	-	-	5	-	5	0.05	2.3
	Divisor		27	Tot. in Min.	7					Divisor	108	Tot. in Min.	15
					14%								36%

TIME SPENT ON MARKING TESTS													
CONTROL						EXPERIMENTAL							
Teacher / Time Interval	1	2	Tot.	Av/ W/T	Time/ W/T	4	5	6	7	8	Tot.	Av/ W/T	Time/ W/T
1 - 15	-	-	-	-	-	-	-	-	-	4	4	0.27	2.0
15 - 30	2	2	4	1	22.5	-	9	1	1	-	11	0.73	16.5
30 - 45	-	-	-	-	-	-	4	2	-	-	6	0.4	15.0
45 - 60	-	-	-	-	-	-	-	-	-	-	-	-	-
over 60	-	-	-	-	-	4	-	2	-	-	6	0.4	36.0
		Divisor	4	Tot. in Min.	23				Divisor		15	Tot. in Min.	70

CLASS SPENT THIS TIME AT SEAT WORK													
CONTROL						EXPERIMENTAL							
Teacher / Time Interval	1	2	Tot.	Av/ Clss	Time/ Cl.	4	5	6	7	8	Tot.	Av/ Class	Time/ Cl.
1 - 10	4	-	4	0.15	0.8	-	-	-	-	1	1	0.01	0.1
10 - 20	1	2	3	0.11	1.7	-	-	-	-	1	1	0.01	0.2
20 - 30	5	2	7	0.26	6.5	4	-	-	-	5	9	0.08	2.0
30 - 40	-	2	2	0.07	2.5	14	24	10	6	-	54	0.5	17.5
40 - 50	-	-	-	-	-	6	-	-	8	3	17	0.16	7.2
		Divisor	27	Tot. in Min.	12				Divisor		108	Tot. in Min.	27
					24%								64%

TIME SPENT ON RECORD KEEPING													
CONTROL						EXPERIMENTAL							
Teacher / Time Interval	1	2	Tot	Av/ W/T	Time/ W/T	4	5	6	7	8	Tot.	Av/ W/T	Time/ W/T
1 - 15	4	2	6	1.5	11.3	2	5	1	4	5	17	1.13	8.5
15 - 30	-	-	-	-	-	5	6	-	-	-	11	0.73	16.5
30 - 45	-	-	-	-	-	-	5	-	-	-	5	0.33	12.4
45 - 60	-	-	-	-	-	-	-	1	-	-	1	0.07	3.7
over 60	-	-	-	-	-	-	-	-	-	-	-	-	-
		Divisor	4	Tot. in Min.	11					Divisor	15	Tot. in Min.	41

TIME SPENT ON SUPERVISING TEST (QUIZ)													
CONTROL						EXPERIMENTAL							
Teacher / Time Interval	1	2	Tot.	Av/ Class	Time/ Cl.	4	5	6	7	8	Tot.	Av/ Cl.	Time/ Cl.
1 - 10	-	-	-	-	-	-	-	4	-	1	5	0.05	0.3
10 - 20	-	-	-	-	-	-	-	-	-	-	-	-	-
20 - 30	-	-	-	-	-	-	-	-	-	-	-	-	-
30 - 40	1	1	2	0.07	2.5	-	-	-	3	-	3	0.03	1.1
40 - 50	1	-	1	0.04	1.8	-	-	-	-	-	-	-	-
		Divisor	27	Tot. in Min.	4					Divisor	108	Tot. in Min.	1
					8%								2%

TIME SPENT IN DEPARTMENTAL MEETINGS													
	CONTROL					EXPERIMENTAL							
Time Interval \ Teacher	1	2	Tot.	Av/ W/T	Time/ W/T	4	5	6	7	8	Tot.	Av/ W/T	Time/ W/T
1 - 15	-	-	-	-	-	-	-	-	-	1	1	0.07	0.5
15 - 30	-	-	-	-	-	-	-	1	2	2	5	0.33	7.4
30 - 45	-	-	-	-	-	-	3	1	-	1	5	0.33	12.4
45 - 60	-	1	1	0.25	13.1	-	-	-	-	-	-	-	-
over 60	-	-	-	-	-	-	-	1	1	-	2	0.13	11.7
	Divisor		4	Tot. in Min.	13					Divisor	15	Tot. in Min.	32

CLASS SPENT THIS TIME AT WORKING SAMPLE AT SEAT/BOARD													
	CONTROL					EXPERIMENTAL							
Time Interval \ Teacher	1	2	Tot.	Av/ Cl.	Time/ Cl.	4	5	6	7	8	Tot.	Av/ Cl.	Time/ Cl.
1 - 10	-	-	-	-	-	-	-	-	-	-	-	-	-
10 - 20	-	-	-	-	-	-	16	-	-	-	16	0.15	2.3
20 - 30	-	-	-	-	-	-	-	-	-	-	-	-	-
30 - 40	-	-	-	-	-	-	-	-	-	-	-	-	-
40 - 50	-	-	-	-	-	-	-	-	-	-	-	-	-
	Divisor		27	Tot. in Min.	-					Divisor	108	Tot. in Min.	2
					0%								5%

TIME SPENT IN GROUP PLANNING													
CONTROL						EXPERIMENTAL							
Time Interval \ Teacher	1	2	Tot.	Av/ W/T	Time/ W/T	4	5	6	7	8	Tot.	Av/ W/T	Time/ W/T
1 - 15	-	-	-	-	-	4	3	-	-	8	15	1	7.5
15 - 30	-	-	-	-	-	-	-	-	-	-	-	-	-
30 - 45	-	-	-	-	-	-	-	3	-	-	3	0.2	7.5
45 - 60	-	-	-	-	-	-	-	-	-	-	-	-	-
over 60	-	-	-	-	-	-	-	-	-	-	-	-	-
		Divisor	4	Tot. in Min.	0					Divisor	15	Tot. in Min.	15

TIME SPENT ON TAKING UP HOMEWORK													
CONTROL						EXPERIMENTAL							
Time Interval \ Teacher	1	2	Tot.	Av/ Cl.	Time/ Cl.	4	5	6	7	8	Tot.	Av/ Cl.	Time/ Cl.
1 - 10	11	3	14	0.52	2.6	-	-	-	-	-	-	-	-
10 - 20	2	2	4	0.15	2.3	-	-	-	-	-	-	-	-
20 - 30	1	2	3	0.11	2.8	-	-	-	-	-	-	-	-
30 - 40	-	-	-	-	-	-	-	-	-	-	-	-	-
40 - 50	-	-	-	-	-	-	-	-	-	-	-	-	-
		Divisor	27	Tot. in Min.	8					Divisor	108	Tot. in Min.	0
					16%								0%

TIME SPENT ON MARKING ASSIGNMENTS													
CONTROL						EXPERIMENTAL							
Time Interval \ Teacher	1	2	Tot.	Av/ W/T	Time/ W/T	4	5	6	7	8	Tot.	Av/ W/T	Time/ W/T
1 - 15	-	-	-	-	-	-	-	-	1	-	1	0.07	0.5
15 - 30	2	2	4	1	22.5	-	7	-	-	-	7	0.47	10.6
30 - 45	-	-	-	-	-	-	5	-	-	-	5	0.33	12.4
45 - 60	-	1	1	0.25	13.1	-	1	-	-	-	1	0.07	3.7
over 60	-	-	-	-	-	-	-	-	-	-	-	-	-
		Divisor	4	Tot. in Min.	36					Divisor	15	Tot. in Min.	27

TIME SPENT ON TEACHING ENTIRE CLASS													
CONTROL						EXPERIMENTAL							
Time Interval \ Teacher	1	2	Tot.	Av/ Cl.	Time/ Cl.	4	5	6	7	8	Tot.	Av/ Cl.	Time/ Cl.
1 - 10	-	-	-	-	-	20	16	10	7	4	57	0.53	2.7
10 - 20	6	1	7	0.26	3.9	2	-	-	-	2	4	0.04	0.6
20 - 30	5	4	9	0.33	8.3	-	-	-	-	1	1	0.01	0.3
30 - 40	1	-	1	0.04	1.4	-	8	2	-	-	10	0.09	3.2
40 - 50	-	4	4	0.15	6.8	-	-	-	-	-	-	-	-
		Divisor	27	Tot. in Min.	20					Divisor	108	Tot. in Min.	7
					40%								17%

TIME SPENT ON PREPARING AIDS													
CONTROL						EXPERIMENTAL							
Teacher / Time Interval	1	2	Tot	Av/ W/T	Time/ W/T	4	5	6	7	8	Tot.	Av/ W/T	Time/ W/T
1 - 15	-	-	-	-	-	-	-	-	-	-	-	-	-
15 - 30	-	-	-	-	-	-	-	1	-	-	1	0.07	1.6
30 - 45	-	1	1	0.25	9.4	-	2	-	-	-	2	0.13	4.9
45 - 60	-	-	-	-	-	-	2	-	-	-	2	0.13	6.8
over 60	-	-	-	-	-	-	-	-	-	-	-	-	-
	Divisor		4	Tot. in Min.	9				Divisor		15	Tot. in Min.	13

TIME SPENT ON TEACHING DIFFERENT ABILITY GROUPS													
CONTROL						EXPERIMENTAL							
Teacher / Time Interval	1	2	Tot.	Av/ Cl.	Time/ Cl.	4	5	6	7	8	Tot.	Av/ Cl/	Time/ Cl.
1 - 10	11	-	11	0.41	2.1	2	-	-	-	-	2	0.02	0.1
10 - 20	-	-	-	-	-	-	16	-	-	-	16	0.15	2.3
20 - 30	-	-	-	-	-	-	-	-	-	-	-	-	-
30 - 40	-	-	-	-	-	2	8	-	-	-	10	0.09	3.2
40 - 50	-	-	-	-	-	-	-	-	-	-	-	-	-
	Divisor		27	Tot. in Min.	2				Divisor		108	Tot. in Min.	6
					4%								14%

TIME SPENT ON PLANNING LESSON													
CONTROL						EXPERIMENTAL							
Time Interval \ Teacher	1	2	Tot.	Av/ W/T	Time/ W/T	4	5	6	7	8	Tot.	Av/ W/T	Time/ W/T
1 - 15	10	5	15	3.75	28	9	-	5	-	5	19	1.27	9.5
15 - 30	-	2	2	0.5	11	3	9	-	-	-	12	0.8	18.0
30 - 45	-	-	-	-	-	-	-	-	-	-	-	-	-
45 - 60	-	-	-	-	-	-	-	-	-	-	-	-	-
over 60	-	-	-	-	-	-	-	-	-	-	-	-	-
		Divisor	4	Tot. in Min.	39					Divisor	15	Tot. in Min.	28

TIME SPENT ON TEACHING SMALL GROUPS (3 - 6)													
CONTROL						EXPERIMENTAL							
Time Interval \ Teacher	1	2	Tot.	Av/ Cl.	Time/ Cl.	4	5	6	7	8	Tot.	Av/ Cl.	Time/ Cl.
1 - 10	6	1	7	0.26	1.3	4	24	-	-	3	31	0.29	1.5
10 - 20	-	-	-	-	-	-	-	6	-	-	6	0.06	0.9
20 - 30	-	-	-	-	-	-	-	-	-	-	-	-	-
30 - 40	-	-	-	-	-	-	-	-	6	-	6	0.06	2.1
40 - 50	-	-	-	-	-	-	-	-	-	-	-	-	-
		Divisor	27	Tot. in Min.	1					Divisor	108	Tot. in Min.	5
					2%								12%

APPENDIX G

TABULATION OF RESPONSES TO THE QUESTIONNAIRE

	CONTROL		EXPERIMENTAL				
Teacher / Quest.	1	2	4	5	6	7	8
1	Y	Y	N	N	N	N	N
2	N	N	Y	Y	Y	Y	Y
3	N	N	Y	Y	Y	Y	Y
4	N	N	N	N	N	N	N
5	Y	N	N	N	N	N	N
6	N	N	Y	Y	N	N	Y
7	Y	Y	Y	N	Y	N	Y
8	N	N	N	Y	N	Y	N
9	N	N	Y	Y	N	Y	N
10	N	N	N	N	N	N	N
11	N	Y	Y	Y	N	N	N
12	N	N	Y	Y	Y	Y	N
13	N	N	N	N	N	N	N
14	N	N	N	Y	N	Y	Y
15	Y	Y	N	N	Y	N	N
16	N	N	Y	Y	Y	N	Y
17	Y	N	Y	Y	Y	Y	Y
18	N	N	Y	Y	N	N	Y
19	N	N	N	N	N	N	Y
20	N	N	N	N	N	N	N
21	Y	Y	N	N	N	N	Y
22	N	N	Y	N	Y	Y	Y
23	Y	Y	N	N	Y	N	N
24	N	N	Y	Y	Y	Y	Y
25	N	N	Y	Y	Y	Y	Y
26	-		e	e	e	e	e
27	-		Y	Y	Y	Y	Y
28	-		N	Y	N	Y	Y
29	-		N	N	N	N	N
30			Y	Y	Y	Y	Y
31			Y	Y	Y	N	N
32			N	Y	N	N	N
33			bet.	equ. well	bet.	bet.	bet.
34			fas.	as quic	fas.	fas.	fas.
35			Y	Y	Y	Y	Y
36			b	b	a	b	b
37			Y	Y	Y	Y	Y
38			N	Y	N	N	Y
39			N	Y	Y	Y	Y

	CONTROL		EXPERIMENTAL				
Teacher / Ques.	1	2	4	5	6	7	8
40			N	N	Y	N	Y
41			N	Undec.	N	N	N
42			Y	Undec	Y	Y	Y
43			N	Y	Und.	N	Y
44			Y	Y	Y	Y	Y
45			Y	Y	Y	Y	Y
46			Y	N	Y	Y	Y
47			Y	N	Y	Y	Y
48			Y	Y	Y	N	Y
49			Y	N	Y	Y	N
50			N	Y	Y	N	N
51			N	N	N	Y	N
52			N	Y	N	Y	N
53			a	c	a	c	a
54							
55							